

IDENTIFICATION OF SHIP MODEL
MOTION PARAMETERS

by

Jorge Manuel Delgado Beirao Reis

IDENTIFICATION OF SHIP MODEL
MOTION PARAMETERS

by

JORGE MANUEL DELGADO BEIRÃO REIS
B. S., ESCOLA NAVAL (PORTUGAL)
(1966)

SUBMITTED IN PARTIAL FULFILLMENT
OF THE REQUIREMENTS FOR THE
DEGREE OF NAVAL ENGINEER
AND THE DEGREE OF MASTER
OF SCIENCE IN
MECHANICAL ENGINEERING

at the
MASSACHUSETTS INSTITUTE OF
TECHNOLOGY

June 1971

1. Thesis
2. Introduction
3. Literature Review
4. Methodology
5. Results
6. Discussion
7. Conclusion
8. References

IDENTIFICATION OF SHIP MODEL MOTION PARAMETERS

By

JORGE MANUEL DELGADO BEIRÃO REIS

Submitted to the Department of Naval Architecture and Marine Engineering and the Department of Mechanical Engineering in partial fulfillment of the requirements for the degrees of Naval Engineer and Master of Science in Mechanical Engineering

ABSTRACT

The applicability of the methods of system identification to the determination of ship model motion parameters is studied taking as an example a model of the Mariner class hull form.

A mathematical expression describing the dynamical behaviour of an ocean vehicle is first presented. Then the values of the ship model motion parameters to be identified are predicted using available techniques. Data representing the dynamical behaviour of the hull form under controlled conditions is obtained with model tests in the towing tank. This data is then used together with the first estimated values of the motion parameters to determine the corrected values of those parameters.

The sensitivity of convergence to several noise factors characteristic of the process is also studied.

The results indicate a fairly strong dependence on those noise factors and on the initial uncertainty that in general decide the final convergence. This fact seems to indicate that the particular technique used and the particular input adopted are not in conjunction the most appropriate for this type of dynamical behaviour.

Thesis Supervisor : Martin A. Abkowitz
Title : Professor of Naval Architecture

ACKNOWLEDGMENTS

The author wishes to extend his sincere thanks to his thesis supervisor, Professor Martin A. Abkowitz, for his assistance throughout the project. Appreciation is also expressed to Professor David N. Wormley for his acceptance in acting as the thesis reader for the Department of Mechanical Engineering.

The help of Mr. Ysabel Mejia of the M.I.T. Shipmodel Towing Tank in conducting the experiments and of Mr. Emile Boulos of the Charles Draper Laboratories in preparing equipment and circuits for the measurements is also sincerely appreciated.

The assistance of Mr. Thomas Deville, Mr. Michael Hayes and Mr. Fohn Odea deserve also a special thanks.

Mrs. Barbara Burke, who typed the manuscript, is also entitled to the author's appreciation.

The model tests were run at the M.I.T. Shipmodel Towing Tank.

The analysis of the data was performed at the Mechanical Engineering Computation Center and at the M.I.T. Information Processing Center.

TABLE OF CONTENTS

<u>DESCRIPTION</u>	<u>PAGE</u>
Title Page.....	1
Abstract.....	2
Acknowledgments.....	3
Table of Contents.....	4
List of Figures.....	5
Introduction.....	6
Equations of Motion.....	13
Motion Parameters.....	22
Excitation from the Environment.....	37
Experimental Procedure.....	48
Identification of Ship Model Motion Parameters.....	63
Discussion of Results.....	75
Conclusions and Recommendations.....	88
Bibliography.....	93
Appendices:	
1. Hydrostatic Coefficients.....	97
2. Linear Hydrodynamic Coefficients.....	117
3. Non-linear Motion Parameters.....	145
4. Circuitry and Digitizing Program.....	151
5. Identification Procedure.....	155

LIST OF FIGURES

<u>FIGURE</u>	<u>DISCRIPTION</u>	<u>PAGE</u>
2.1	Orientation of the coordinate systems	13
2.2	Relationship between \dot{z}_0 , w and ϕ	17
5.1	Rate-Gyro excitation circuit	54
5.2	AC/DC conversion of gyro output	55
A1.1	Hydrostatic force as a function of z_0 and ϕ	114
A1.2	Hydrostatic moment as a function of z_0 and ϕ	115
A3.1	Symbols for the calculation of the non linear damping coefficient	149
A4.1	Example of patching the 680 analog computer	154
A5.1	Typical behaviour of the parameters	181

CHAPTER 1

INTRODUCTION

Consider a system whose inputs are known and the outputs observable. System identification, in general, has the objective of determining the structure of that system using the known inputs and measuring the outputs in order that one may be able to predict the behaviour of the system (outputs) to the various excitations expected from the environment (inputs). One such a system of interest in naval architecture is the ocean vehicle. The naval architect wants to be able to predict the velocities, accelerations and other important motion responses that a ship will experiment at sea in order to effectively design for proper response, structural integrity, and other critical aspects of operability.

The identification of a system of completely unknown structure is, in most cases, impossible, at least for the time being. There are, however, some types of systems for which identification is possible. Those consist of parameter identification problems where the general structure of system is known but the particular values for a set of parameters are to be determined. The ocean vehicle has, fortunately, these characteristics. Using rigid body dynamics, one is able to develop the so-called equations of motion which describe the dynamical behaviour of the

vehicle. In these equations there are some parameters (ship motion parameters or hydrodynamic coefficients) which are not known or at best whose value is known with a certain degree of uncertainty. It will be necessary then, in order to fully predict the behaviour of ocean vehicles at sea, to obtain the correct values of those parameters or, at least, by selected measurements, to decrease the degree of uncertainty. Unfortunately these measurements are, more often than not, combined with noise from the instruments themselves or from the environment at the time the measurements take place or from both. These types of noise need to be taken into account when identifying the parameters. An example of the foregoing is the case of experiments in a towing tank to obtain data from models to apply to full scale vehicles. Measurement noise exists whenever one uses any type of transducers that always show some hysteresis and are characterized by a specific dynamical behaviour. Environment noise will be caused among others by interference between the model and undesired waves, and vibration of test equipment. Of course these effects could be included in the mathematical model of the vehicle but it would make the analysis of its behaviour much more difficult. Thus it is seen that sometimes it's preferable

to assume a simpler model for the behaviour of the system and introduce the effects not accounted for in the mathematical model as noise (environmental noise) whose general nature is assumed known.

Many of the available techniques of system identification were developed for a dynamical system whose behaviour may be expressed in a so-called state space representation[1]:

$$\dot{\underline{x}} = \underline{f}(\underline{x}, \underline{u}, t) ; \quad \underline{x}(t_0) = \underline{x}_0 \quad (1.1)$$

$$\underline{y} = \underline{h}(\underline{x}, \underline{u}, t) \quad (1.2)$$

where the vector \underline{x} expresses the state of the system at time t ($t > t_0$), the vector \underline{y} represents the output of the system at time t and the vector \underline{u} the input at time t .

\underline{x}_0 is the state of the system at time t_0 (initial conditions)

and $\dot{\underline{x}}$ is the derivative of \underline{x} with respect to time. \underline{f} and

\underline{h} are vectors that describe the structure of the system.

The main assumption to be made at this stage about the

above system in order that meaningful results can be

obtained is that there will be unique values of $\underline{x}(t)$ and

$\underline{y}(t)$ for a given $\underline{u}(t)$ and $\underline{x}(t_0) = \underline{x}_0$ for all t such that

$t > t_0$.

The purpose of any technique of system identification

will then be, by operating on $\underline{u}(t)$ and $\underline{y}(t)$, the deter-

mination of \underline{f} and \underline{h} . As said before it will be, in

general, not possible to obtain the solution to this problem. Therefore, the assumption of a known structure except for a set of unknown parameters is made and the system is expressed by the following:

$$\dot{\underline{x}} = \underline{f}(\underline{x}, \underline{u}, \underline{p}, t) \quad \underline{x}(t_0) = \underline{x}_0 \quad (1.3)$$

$$\underline{y} = \underline{g}(\underline{x}, \underline{u}, \underline{p}, t) \quad (1.4)$$

where the vector \underline{p} represents the set of parameters to be determined.

In order that the identification of those parameters be of some value, it is further assumed that \underline{p} is time invariant in the time interval of interest $t_0 < t < t_1$, or at least that its time dependence is known meaning that once some $\underline{p}(\tau)$, $t_0 < \tau < t_1$, is determined, $\underline{p}(t)$ will be known in the interval $[t_0, t_1]$.

It must be emphasized at this stage that the foregoing does not assume the uniqueness of that representation. Only the uniqueness of the input/output relationship is assumed. That is, there will be for a given $\underline{u}(t_0, t)$ and $\underline{y}(t)$, a unique \underline{p} such that

$$\underline{y}(t) = \underline{p}(\underline{u}(t_0, t), t) \quad (1.5)$$

It will be possible to show the existence of a representation \underline{x}' , \underline{f}' , \underline{h}' and \underline{p}' such that

$$\dot{\underline{x}}' = \underline{f}'(\underline{x}', \underline{u}, \underline{p}', t) \quad \underline{x}'(t_0) = \underline{x}_0' \quad (1.6)$$

$$\underline{y} = \underline{h}'(\underline{x}', \underline{u}, \underline{p}', t) \quad (1.7)$$

The problem is then redefined as the determination of a set of parameters \underline{p} of some state representation of the system defined by $\underline{y}(t) = \underline{h}(\underline{u}(t_0, t), t)$.

Coming back to the case of an ocean vehicle, it will be shown that the resulting equations of motion may be regarded as a state representation of the vehicle dynamical behaviour and that the assumptions made so far are met by that mathematical model.

The foregoing assumes, apart from the unknown set of parameters \underline{p} , no other types of uncertainty. But, as said before, it is sometimes advantageous to select a simpler mathematical model for a given system and introduce the so-called system or environmental noise since it results from interactions between the system and the environment. Also consideration must be given to measurement noise. A model that takes into account those factors will be of the following form [2] :

$$\dot{\underline{x}} = \underline{f}(\underline{x}, \underline{u}, \underline{p}, \underline{w}, t) \quad \underline{x}(t_0) = \underline{x}_0 \quad (1.8)$$

$$\underline{y} = \underline{h}(\underline{x}, \underline{u}, \underline{p}, \underline{y}, t) \quad (1.9)$$

where the vector \underline{w} represents the environmental noise of the system¹ and the vector \underline{y} the measurement noise.

¹Errors in the structure of the system will come through as environmental noise.

In addition (case of time varying systems) the parameter vector \underline{p} may have the following structure:

$$\dot{\underline{p}} = \underline{g}(\underline{p}, t, \underline{w}') \quad \underline{p}(t_0) = \underline{p}_0 \quad (1.10)$$

where \underline{g} is known and \underline{w}' is noise.

In order to simplify the above formulation the following assumptions are generally made:

1. The mathematical model is time invariant.
2. System structural uncertainty will appear as an apparent additive noise process.
3. Measurement noise may also be considered an additive noise process.
4. Output measurements are linear functions of the state of the system and are structurally independent of the input.
5. Parameters are coefficients in the model structure that are to be identified.

The system dynamical behaviour may be expressed as follows:

$$\dot{\underline{x}} = \underline{f}(\underline{x}, \underline{u}, \underline{p}) + \underline{w} \quad (1.11)$$

$$\dot{\underline{p}} = 0 \quad (1.12)$$

$$\underline{y} = \underline{H} \underline{x} + \underline{v} \quad (1.13)$$

where \underline{H} is the measurement matrix, often a diagonal matrix.

The purpose of the identification will be the determination of p.

In Chapter 2 the equations of motion for an ocean vehicle will be developed and it will be shown that indeed they constitute a state space representation of the ocean vehicle's dynamical behaviour. It will be shown also that they satisfy the foregoing assumptions.

In Chapter 3 the methods of predicting the ship model motion parameters will be outlined and values of those parameters for the mariner class hull form will be presented (Appendices 1, 2, 3).

In Chapter 4 the excitation from the environment will be described and its several components isolated.

In Chapter 5 the experimental procedure followed in order to obtain the data will be described.

In Chapter 6 the different methods of system identification will be discussed and those which seem more convenient will be used to identify the motion parameters of the mariner class hull form.

In Chapter 7 the results obtained in Chapter 6 will be discussed and in Chapter 8 the consequent conclusions and recommendations will be pointed out.

CHAPTER 2

EQUATION OF MOTION

In order to develop the equations of motion for an ocean vehicle, it is convenient first to define two rectangular right handed coordinate systems, one fixed in the surface of undisturbed water with the positive $x_3 = z_0$ axis pointing downward and the second moving with the vehicle, the origin of such coordinate system being arbitrary (but the coordinates (x_G, y_G, z_G) of the center of gravity of the vehicle in this system being known) pointing the positive $x_1 = x$ axis to the bow of the vehicle and the $x_3 = z$ pointing downward in the equilibrium position (see Figure 2.1)

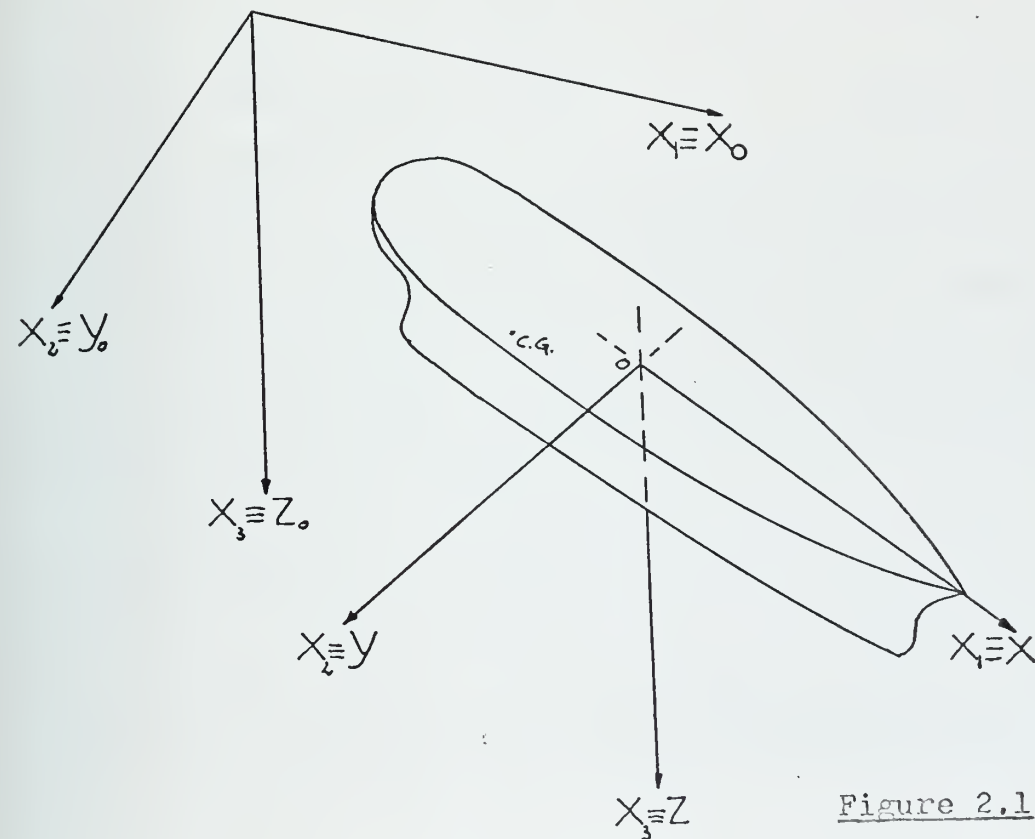


Figure 2.1

The orientation of the moving axis system with respect to the fixed axis system will be expressed by three angles (the "modified Euler angles") defined as follows, assuming that initially both axis systems coincide [3]:

ψ - angle of rotation of the moving axis system about the $z = z_0$ axis (yaw angle)

θ - rotation of the moving coordinate system already yawed about its y axis (pitch angle)

ϕ - rotation of the moving coordinate system (already yawed and pitched) about its x axis (roll angle).

Defining the rotation about the body axis as respectively S_4 (about the body x_1 axis), S_5 (about the body x_2 axis) and S_6 (about the body x_3 axis) the relation between those rotations and the Euler angles will be [3]:

$$\begin{aligned} S_4 &= \phi \cos \theta \cos \psi - \theta \sin \psi \\ S_5 &= \phi \cos \theta \sin \psi + \theta \cos \psi \\ S_6 &= -\phi \sin \theta + \psi \end{aligned} \quad (2.1)$$

The motion of a rigid body in six degrees of freedom with respect to an axis system fixed in space, when the origin is at the center of gravity and the moving axes are the principal axes of inertia, may be represented by Newton's equations, namely:

$$\begin{aligned} \underline{F} &= \frac{d}{dt} (\underline{m} \underline{u}_G) \\ \underline{M} &= \frac{d}{dt} (\underline{I} \underline{\Omega}_G) \end{aligned} \quad (2.2)$$

where \underline{n} is a diagonal matrix whose elements are equal to

the mass of the body, \underline{u}_G is the translational velocity vector of the center of gravity, of components along the moving axes u_G , v_G and w_G , \underline{F} is the total vector force acting on the body of components along the axes X , Y and Z , \underline{I} is a diagonal matrix whose elements are the inertia, $\underline{\Omega}_G$ is the rotational velocity vector about those axes of components p_G , q_G and r_G and \underline{M} is the total moment acting on the body of components K , M , and N . Neglecting the effects due to rotation of the earth in ocean vehicle dynamics Abkowitz [4] and Tufts [5] develop equations (2.2) respectively with fixed and varying mass and position of center of gravity for the case of an arbitrary origin, the first using vector analysis and the second Lagrange's equations. Therefore only the resulting equations will be presented here and for a body of fixed mass and inertia.

$$\begin{aligned} X &= m[\dot{u} + qw - rv - x_G(q^2 + r^2) + y_G(pq - \dot{r}) + z_G(pr + \dot{q})] \\ Y &= m[\dot{v} + ru - pw - y_G(r^2 + p^2) + z_G(qr - \dot{p}) + x_G(qp + \dot{r})] \\ Z &= m[\dot{w} + pv - qu - z_G(p^2 + q^2) + x_G(rp - \dot{q}) + y_G(rq + \dot{p})] \end{aligned} \quad (2.3)$$

$$\begin{aligned} K &= I_{xx}\dot{p} + I_{xy}(\dot{q} - pr) + I_{xz}(\dot{r} + pq) + I_{yz}(q^2 - r^2) + \\ &\quad (I_{zz} - I_{yy})qr - mz_G(\dot{v} - pw + ru) + my_G(\dot{w} - qu + pv) \\ M &= I_{yy}\dot{q} + I_{yz}(\dot{r} - qp) + I_{yx}(\dot{p} + qr) + I_{zx}(r^2 - p^2) + \\ &\quad (I_{yx} - I_{zz})rp - mx_G(\dot{w} - qu + pv) + mz_G(\dot{u} - rv + qw) \end{aligned} \quad (2.4)$$

$$N = I_{zz}\dot{r} + I_{zx}(\dot{p}-rq) + I_{zy}(\dot{q}+rp) + I_{xy}(p^2-q^2) + (I_{yy}-I_{xx})pq-my_G(\dot{u}-rv+qw) + mx_G(\dot{v}-pw+ru) \quad (2.4)$$

where unsubscripted velocities and accelerations refer to an arbitrary origin and x_G , y_G and z_G are the coordinates of the center of gravity.

Note that \underline{F} and \underline{M} will in general be functions of position, velocity and acceleration of the vehicle as well of properties of fluid, geometry of the body and of the inputs. For a body of given geometry and mass distribution and in a given fluid then

$$\frac{\underline{F}}{\underline{M}} = \frac{\underline{F}}{\underline{M}} [\underline{u}, \underline{\Omega}, \dot{\underline{u}}, \dot{\underline{\Omega}}, \underline{x}, \underline{s}, \text{input}]$$

where

$$\begin{aligned} \underline{u} &= [u, v, w] \\ \underline{\Omega} &= [p, q, r] \\ \underline{x} &= [x_0, y_0, z_0] \\ \underline{s} &= [\psi, \theta, \phi] \end{aligned} \quad (2.5)$$

Since the purpose of this investigation was the study of the applicability of system identification techniques to ocean vehicle dynamics and not the identification of parameters of a particular vehicle (parameters imbedded in \underline{F} and \underline{M}) it was decided at this stage to simplify the formulation by restraining the model to one or at most two degrees of freedom. It will

be only considered heave (translation along the z axis) and roll (rotation about the x axis) with zero forward speed. To simplify the formulation even more y_G and z_G were made zero. Therefore, the equations of motion for this restrained mode ($y_G=z_G=\dot{u}=u=\dot{v}=v=\dot{q}=q=\dot{r}=r=x_0=y_0=\theta=0$) will be

$$\begin{aligned} Z &= m\dot{w} \\ K &= I_{xx}\dot{p} \end{aligned} \quad (2.6)$$

and

$$\begin{aligned} Z &= Z[\dot{w}, w, z_0, \dot{p}, p, \phi, \text{input}] \\ K &= K[\dot{w}, w, z_0, \dot{p}, p, \phi, \text{input}] \end{aligned} \quad (2.7)$$

Looking at equations (2.6 and (2.7) one sees that they are almost in a state space form except for the fact that Z and K are functions of \dot{w} and \dot{p} and that

$$p = \frac{d}{dt} [s_4] = \frac{d}{dt} [\phi \cos \theta \cos \psi - \theta \sin \psi] \quad \text{from Eq. (2.1)}$$

$$\dot{z}_0 = w \cos \phi + v \sin \phi \quad \text{see Fig. 2.2}$$

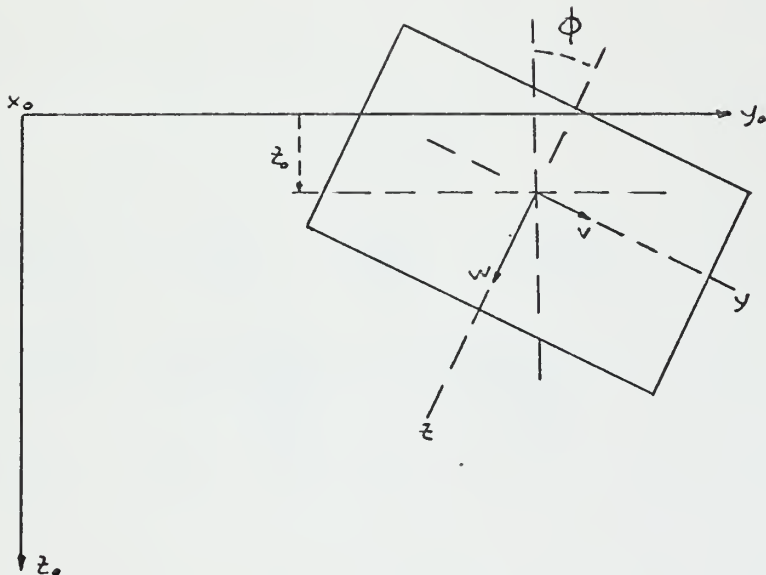


Figure 2.2

Since ψ , θ and v are zero, the above relations reduce to

$$\begin{aligned}\dot{\phi} &= p \\ \dot{z}_0 &= w \cos \phi\end{aligned}\quad (2.8)$$

Ignoring for the moment the presence of \dot{w} and \dot{p} as arguments of Z and K the complete set of equations in state space form would be:

$$\begin{aligned}\dot{w} &= \frac{1}{m} Z(\dot{w}, w, z_0, \dot{p}, p, \phi, \text{input}) \\ \dot{z}_0 &= w \cos \phi \\ \dot{p} &= \frac{1}{I_{xx}} K(\dot{w}, w, z_0, \dot{p}, p, \phi, \text{input}) \\ \dot{\phi} &= p\end{aligned}\quad (2.9)$$

In order to be able to proceed and apply the existing identification techniques to ocean vehicles (specifically surface ships), it is necessary to isolate \dot{w} and that the dependence of Z and K on \dot{w} and \dot{p} is linear. Therefore, the next step will be to assume the existence of a Taylor's expansion about an equilibrium point, to be taken as $\dot{w}=w=0, \dot{p}=p=0$ of the form:

$$\begin{aligned}Z = Z(0) &+ \left. \frac{\delta Z}{\delta \dot{w}} \right|_0 \dot{w} + \left. \frac{\delta Z}{\delta w} \right|_0 w + \left. \frac{\delta Z}{\delta z_0} \right|_0 z_0 + \left. \frac{\delta Z}{\delta \dot{p}} \right|_0 \dot{p} + \left. \frac{\delta Z}{\delta p} \right|_0 p + \left. \frac{\delta Z}{\delta \phi} \right|_0 \phi + \left. \frac{\delta Z}{\delta \text{input}} \right|_0 \text{input} \\ &+ \frac{1}{2!} \left. \frac{\delta^2 Z}{\delta \dot{w}^2} \right|_0 \dot{w}^2 + \frac{1}{2!} \left. \frac{\delta^2 Z}{\delta w^2} \right|_0 w^2 + \dots\end{aligned}\quad (2.10)$$

$$+ \frac{1}{2!} \frac{\delta^2 Z}{\delta w \delta w} \bigg|_0 \dot{w}w + \frac{1}{2!} \frac{\delta^2 Z}{\delta w \delta z} \bigg|_0 \dot{w}z_0 + \frac{1}{2!} \frac{\delta^2 Z}{\delta w \delta p} \bigg|_0 \dot{w}p + \dots + \quad (2.10)$$

and in the same way for K.

It is known from potential theory [5] that for a body moving in an infinite fluid away from the surface, there are no dependence of force or moments on products of accelerations with velocities or accelerations. It will be necessary to accept this assumption for surface vehicles without any other justification, and neglect also all other terms where acceleration is multiplying any other variable.

With the vehicle in equilibrium also $Z(0) = 0$.

Further simplification can be achieved if we consider each element of the expansion, which will be considered in the next chapter. It will be enough to say for the moment that due to port and starboard symmetry (both geometrically and inertial) the vertical motion of the vehicle does not produce any rolling moment, at least to first order, and therefore those terms can be dropped from the equations further simplifying them [4]. Similarly, and to first order only, rolling motions will not produce any vertical force or, in other words:

$$\frac{\delta Z}{\delta r} \bigg|_0 = \frac{\delta Z}{\delta p} \bigg|_0 = \frac{\delta Z}{\delta \phi} \bigg|_0 = \frac{\delta K}{\delta \dot{v}} \bigg|_0 = \frac{\delta K}{\delta \dot{w}} \bigg|_0 = \frac{\delta K}{\delta \dot{z}} \bigg|_0 = 0 \quad (2.11)$$

Also the square terms $\frac{\delta^2 Z}{\delta w^2}$ and $\frac{\delta^2 K}{\delta p^2}$ will be nonexistent.

Instead, consideration must be given to frictional and vortical effects which are of the form $c_1 w|w|$ and $c_2 p|p|$. Although these terms are not analytic, they will be included so that the model represents a closer approximation to the vehicle physical behaviour.

The form of the equations will then be:

$$\begin{aligned} \dot{w} &= \frac{1}{-Z_{\dot{w}} + n} \left[Z_{\dot{w}} w + Z_{w|w|} w|w| + Z_{z_0} z_0 + \dots + \frac{1}{(m+n)!} Z_{x_i^n x_j^m} (x_i)^n (x_j)^m + \right. \\ &\quad \left. + \text{input} \right] \\ \dot{z}_0 &= w \cos \phi \\ \dot{p} &= \frac{1}{-K_{\dot{p}} + i} \left[K_{\dot{p}} p + K_{p|p|} p|p| + \dots + \frac{1}{(m+n)!} K_{x_j^n x_i^m} (x_j)^n (x_i)^m + \right. \\ &\quad \left. + \text{input} \right] \\ \dot{\phi} &= p \end{aligned} \tag{2.12}$$

$$\text{where } Z_{x_j} = \frac{\delta Z}{\delta x_j} \quad \text{and } K_{x_i} = \frac{\delta K}{\delta x_i}$$

In the next chapter each term in the above equations which is of first order, quadratic or at most cubic will be discussed in relation to its nature, order of magnitude and methods of predicting its value.

In summary, from Newton's equations and for a case of restricted motions, assuming the existence of a Taylor series for the force and moment and linear dependence of

force and moment on acceleration it was possible to obtain a set of equations in a state space representation to which some system identification techniques may be applicable.

CHAPTER 3

MOTION PARAMETERS

For convenience of presentation, the discussion on the motion parameters will be subdivided into three main groups, namely:

i) motion parameters resultant of differentiation of force or moment on the body with respect to position (z_0, ϕ) only (hydrostatic coefficients)

ii) linear terms resultant from differentiation of force or moment on the body with respect to acceleration and velocity (added mass or added moment of inertia and damping coefficients)

iii) non-linear hydrodynamic terms

3.1- Hydrostatic Coefficients

These coefficients are dependent only on the geometry and mass distribution of the body, for example, shape and relative position of the metacenter, center of buoyancy and center of gravity, for the restoring moments.

The coefficients are calculated based on the lines plan of the hull form.

With the ship in the upright position, the buoyant force $B(z_0, 0)$ is calculated for each waterline (each

selected value of z_0). Then at each waterline an angle of roll is simulated in the body plan, being the point of rotation the assumed position of the center of gravity, and the resultant buoyant force $B(z_0, \phi)$ and the righting arm $GZ(z_0, \phi)$ are calculated (for details see, for example, "Principles of Naval Architecture" edited by J. P. Comstock, Chapter 2). The righting moment ($-K(z_0, \phi)$) is then given by the product of $B(z_0, \phi) \cdot GZ(z_0, \phi)$. The vertical force, positive downwards, is the difference between the weight of the body and the buoyant force, or in other words, considering the component of the force along the body x_3 axis. We have:

$$\begin{aligned} Z(z_0, \phi) &= (W - B(z_0, \phi)) \cos \phi \\ K(z_0, \phi) &= -B(z_0, \phi) \cdot GZ(z_0, \phi) \end{aligned} \quad (3.1)$$

where W is the weight of the vehicle.

The values of $Z(z_0, \phi)$ and $K(z_0, \phi)$ are usually presented tabulated or in graphical form (see Appendix 1). In order to get the hydrostatic coefficients, derivatives of (3.1), it is necessary to obtain a mathematical representation of those surfaces, for example, a polynomial fitting by the method of least squares. One such scheme was developed by Kerwin [6] and implemented by Parissis [7] using Legendre Polynomials to represent ship forms which is equally applicable to (3.1).

If one is interested in using in the state equations just the linear terms such a procedure is not necessary.

It is easily seen that

$$\left. \frac{\delta Z(z_0, \phi)}{\delta \phi} \right|_{z_0=\phi=0} = \left. \frac{\delta K(z_0, \phi)}{\delta z_0} \right|_{z_0=\phi=0} = 0 \quad (3.2)$$

due to port-starboard symmetry of the hull form and that

$$\left. \frac{\delta Z(z_0, \phi)}{\delta z_0} \right|_{z_0=\phi=0} = -\rho g A_{WL} \quad (3.3)$$

$$\left. \frac{\delta K(z_0, \phi)}{\delta} \right|_{z_0=\phi=0} = -W \cdot GM$$

where A_{WL} represents the area of the waterplane at the design waterline ($z_0=0$) and GM is the metacentric height (distance from the metacenter to the center of gravity).

Due also to port and starboard symmetry, in the non-linear formulation the term $\frac{1}{2!} \cdot \left. \frac{\delta^2 K(z_0, \phi)}{\delta^2} \right|_{z_0=\phi=0}$ is

equal to zero. Therefore in this case the cubic term will be maintained in the equations.

3.2 - Added Mass, Added Moment of Inertia and Damping Coefficients

From the eight terms that form this subgroup four will be zero again due to symmetry of the vehicle. Those will be the cross coupling terms

$$-Z_p = Z_{\dot{p}} = -K_{\dot{w}} = -K_w = 0 \quad (3.4)$$

and the terms to be retained:

$$\begin{aligned}
 & -Z_{\dot{w}} \\
 & -Z_w \\
 & -K_{\dot{p}} \\
 & -K_p
 \end{aligned}
 \tag{3.5}$$

So it is seen that when only linear terms are considered the equations of heave and roll are completely decoupled and may be studied independently. If the six degrees of freedom were considered, the existence of port and starboard symmetry would cause, when retaining only the linear terms in the equations of motion, the decoupling of the equations into two groups [3,4]:

coupled surge (translation along x_1 axis), heave (translation along the x_3) and pitch (rotation about the x_2 axis) - the so-called motions in the vertical plane, and coupled sway (translation along the x_2 axis), roll (rotation about the x_1 axis) and yaw (rotation about the x_3 axis) in the horizontal plane.

Added mass and damping coefficients are in general not known except for some special types of idealized forms and when the vehicles are excited into some special types of motions. One such type that have been studied quite extensively, especially in the domain of surface vehicles or vehicles near the surface, is that of a

vehicle excited by waves into an oscillatory motion. The path usually followed [3,8,9] is to assume the fluid to be inviscid, incompressible and irrotational, therefore assuring the existence of a velocity potential that satisfies both the Laplace equation and the Bernoulli equation. Considering the motion harmonic, the solution is sought as the product $\text{Re} [\varphi(x_1, x_2, x_3) e^{-i\omega t}]$ where ω is the frequency of oscillation. Considering the linearized problem, $\varphi(x_1, x_2, x_3)$ is assumed to be the sum of eight other potentials being the total motion the superposition of each due to each of the potentials: two of them due to the action of the incident wave plus diffracted wave on a fixed vehicle and six due to forced oscillations of the vehicle on calm water. The fluid is assumed in all cases to extend to $\pm\infty$ in the x_1 and x_2 directions and between $x_3=0$ and $+\infty$ or between $x_3=0$ and $x_3=h$ for the finite depth case. The linearized boundary conditions will be:

a) On the surface $x_3=0$, the velocity of the fluid $\frac{\delta \Phi}{\delta x_3} = \frac{\delta \xi}{\delta t}$ where ξ is the surface elevation, for all the potentials and is equal to $\frac{-\omega^2}{g} \varphi_j$ using the linearized Bernoulli equation.

b) On the body for the incident and diffracted

potentials $\frac{\delta \varphi}{\delta n} = 0 = \frac{\delta}{\delta n}(\varphi_w + \varphi_d)$ where n is the unit normal.

For the six other potentials the normal component of the velocity will equal the velocity of the body in

that direction or $R \left[\frac{\delta \varphi_j}{\delta n} e^{-i\omega t} \right] = f_j \dot{s}_j$ or being

$$\dot{s}_j = R \left[i\omega s_{ja} e^{-i\omega t} \right] \quad (s_{ja} \text{ is the amplitude of the oscillation in the } j \text{ direction})$$

$$\frac{\delta \varphi_j}{\delta n} = -i\omega f_j s_{ja} \quad (3.6)$$

where f_j ($j = 1, \dots, 6$) are the generalized direction cosines of the body surface

$$f_j \quad (j=1,2,3) = \cos(n, x_j)$$

$$f_4 = x_2^f x_3^f - x_3^f x_2^f$$

$$f_5 = x_3^f x_1^f - x_1^f x_3^f$$

$$f_6 = x_1^f x_2^f - x_2^f x_1^f$$

c) At

$$x_3 = +\infty \quad \nabla \Phi = 0$$

$$x_3 = +h \quad \frac{\delta \Phi}{\delta n} = 0 \quad (3.7)$$

d) At $r = (x_1^2 + x_2^2)^{\frac{1}{2}}$ the radiation condition must be satisfied for the six potentials and for diffraction potential or:

$$\lim_{r \rightarrow \infty} r^{\frac{1}{2}} \left[\frac{\delta \varphi_j}{\delta r} + i \frac{\omega^2}{g} \varphi_j \right] = 0 \quad j = 1 \dots 6, d \quad (3.8)$$

Also from Bernoulli equation the hydrodynamic pressure will be

$$p = -\rho \frac{\delta \Phi}{\delta t} = \text{Re} [i\omega \rho \varphi e^{-i\omega t}] = \text{Re} \left[i\omega \rho [\varphi_w + \varphi_d + \sum_{j=1}^6 \varphi_j] e^{-i\omega t} \right] \quad (3.9)$$

The hydrodynamic force in the k direction will be

$$F_k = -\iint_{S_0} p f_k ds = \text{Re} \left[-i\omega \rho e^{-i\omega t} \iint_{S_0} [\varphi_w + \varphi_d + \sum_{j=1}^6 \varphi_j] f_k ds \right] \quad (3.10)$$

The total hydrodynamic force on the body in a given direction k was defined to be in Chapter 2 for motions.

$$\text{in mode } j: F_{kj} = \frac{\delta F_{kj}}{\delta s_j} s_j + \frac{\delta F_{kj}}{\delta \dot{s}_j} \dot{s}_j$$

$$\text{being } \dot{s}_j = \text{Re} [-i\omega s_{ja} e^{-i\omega t}]$$

$$s_j = \text{Re} [-\omega^2 s_{ja} e^{-i\omega t}]$$

and changing the nomenclature to

$$\frac{\delta F_{kj}}{\delta s_j} = -A_{kj}, \quad \frac{\delta F_{kj}}{\delta \dot{s}_j} = -B_{kj}$$

$$F_{kj} = \text{Re} [(\omega^2 A_{kj} + i\omega B_{kj}) s_{ja} e^{-i\omega t}] \quad (3.11)$$

and the total force in the k direction was

$$F_k = \sum_{j=1}^6 F_{kj} + F_{kw}$$

F_{kw} being the excitation in the k direction (the hydrostatic force was excluded from the foregoing expression as it was from the Bernoulli equation (3.9)).

Comparing the foregoing expression with (3.10), the following is obtained:

$$F_{wk} = \operatorname{Re} \left[-i\omega\rho e^{-i\omega t} \iint_{S_0} (\varphi_w + \varphi_d) f_k ds \right] \quad (3.12)$$

$$(\omega^2 A_{kj} + i\omega B_{kj}) s_{ja} = -i\omega\rho \iint_{S_0} \varphi_j f_k ds \quad (3.13)$$

and therefore we obtain

$$A_{kj} = \frac{\rho}{s_{ja}\omega} \iint_{S_0} \operatorname{Im} [\varphi_j] f_k ds \quad (3.14)$$

$$B_{kj} = -\frac{\rho}{s_{ja}} \iint_{S_0} \operatorname{Re} [\varphi_j] f_k ds$$

that would give the value of the hydrodynamic coefficients if the velocity potential was known. Since that is not the case, it is not possible to use the above equations. Instead one looks for possible relationships between the hydrodynamic coefficients to get a better insight on the solution of the problem.

For example using the second form of Green's theorem to a velocity potential φ_j and to another velocity potential φ_k with $k \neq j$ we obtain since both potentials satisfy Laplace's equation: [3]

$$\begin{aligned} A_{kj} &= A_{jk} \\ B_{kj} &= B_{jk} \end{aligned} \quad (3.15)$$

Also from [9] and [10] the following expression is obtained relating added mass and damping coefficients:

$$A_{kj}(\omega) - A_{kj}(\infty) = \frac{2}{\pi} \int_0^{\infty} [B_{kj}(\xi) - B_{kj}(\infty)] \frac{\xi d\xi}{\xi^2 - \omega^2} \quad (3.16)$$

$$B_{kj}(\omega) - B_{kj}(\infty) = \frac{2}{\pi} \int_0^{\infty} [A_{kj}(\xi) - A_{kj}(\infty)] \frac{\xi^2 d\xi}{\xi^2 - \omega^2}$$

Maybe the most important relations are the so-called Haskind relations [11] based also on the second form of Green's theorem when applied to equation (3.12) after the expression for f_k in (3.6) or:

$$f_k = -\frac{\delta \mathcal{P}_k}{\delta n} \frac{1}{i\omega s_{ka}} \quad (3.6)$$

is substituted in (3.12):

$$F_{wk} = \text{Re} \left[\frac{\mathcal{P}}{s_{ka}} e^{-i\omega t} \iint_{s_0} (\mathcal{P}_w + \mathcal{P}_d) \frac{\delta \mathcal{P}_k}{\delta n} ds \right] \quad (3.12)$$

Green's theorem

$$\iiint_R (\varphi^2 \Delta \psi - \psi \Delta \varphi^2) dr = \oint_{s_0} \left(\varphi \frac{\delta \psi}{\delta n} - \psi \frac{\delta \varphi}{\delta n} \right) ds \quad (3.13)$$

for $\varphi = \mathcal{P}_d$
 $\psi = \mathcal{P}_k$

Since both potentials satisfy Laplace's equation the left hand side is zero and therefore:

$$\iint_{s_0} \mathcal{P}_d \cdot \frac{\delta \mathcal{P}_k}{\delta n} ds = \iint_{s_0} \mathcal{P}_k \frac{\delta \mathcal{P}_d}{\delta n} ds \quad (3.14)$$

and from the boundary condition on the body

$$\frac{\delta \mathcal{P}_d}{\delta n} = - \frac{\delta \mathcal{P}_w}{\delta n} \quad (3.15)$$

The following will result for the excitation force:

$$F_{wk} = \operatorname{Re} \left[\frac{\rho}{s_{ka}} e^{-i\omega t} \iint_{S_0} \left[\varphi_w \frac{\delta \varphi_k}{\delta n} - \varphi_k \frac{\delta \varphi_w}{\delta n} \right] ds \right] \quad (3.16)$$

eliminating the diffraction potential.

Following [11] for 2-dimensional bodies (under beam seas) the following relations are obtained, based on (3.16):

$$F_{kk} = \frac{\omega}{\rho g^2 \beta_a^2} \left| F_{ka} \right|^2 \quad (3.17)$$

and therefore

$$F_{ka} = \beta_a \left[\frac{\rho g^2}{\omega} B_{kk} \right]^{\frac{1}{2}} \quad k = 2, 3, 4$$

where β_a is the amplitude of the incoming waves and F_{ka} is the amplitude of the exciting force. Being the damping coefficient calculated from the energy radiated at infinity it's possible to calculate the magnitude of the exciting force. However, it's phase is not determined.

From the foregoing it is seen that the analytic solution for the general case of a three dimensional body does not exist. Only the cases of the spheres and ellipsoids have been solved [12]. It would be possible to solve a problem numerically by such a procedure as the finite element technique [3], but nothing is available yet.

In order to get information for practical applications it is necessary then to work into two dimensional problems. The strip theory for ship motions, introduced by Korvin-Kroukovsky [13, 14, 15] makes use of two dimensional solutions for each transverse section of the body

determining the characteristics on the flow and consequently the local hydrodynamic coefficients and then the results are integrated over the length of the body to obtain the hydrodynamic coefficients for the whole vehicle, applying then a correction factor to take into account the existence of a three dimensional flow.

The solution in two dimensions has been obtained for the three possible modes of motion: sway, heave and roll. One way to obtain the solution is, using conformal mapping to transform the cross section of arbitrary shape of a two dimensional cylinder into the unit circle, to assume the velocity potential to be a sum of multipole potentials whose coefficients are adjusted to satisfy the boundary conditions on the cylinder [16]. For example for heave the multipole expansion will be formed by the sum of a source potential at the origin plus the multipole potentials while for sway and roll instead of a source there will be a dipole since the heave potential is symmetrical and the other two asymmetrical.

The conformal transformation will be of the form [8]

$$\frac{z}{a} = \zeta + \sum_{n=0}^N a_{2n+1} \zeta^{-(2n+1)} \quad (3.18)$$

which maps an arbitrary section in the z plane into the unit circle in the ζ plane. From the section offsets it is necessary to find the coefficients a_{n+1} . A method of determining those coefficients was developed by Bermejo [17] using Theodorsen's method.

Another way of solving the two dimensional problem is to represent the velocity potential by a distribution of sources on the contour of the section [18]. The intensity of the sources is determined applying the kinematic boundary condition to the underwater portion of the section. Once the potential is obtained the pressure is calculated using equation (3.9) - linearized Bernoulli equation - and the force is obtained by integration from where the added mass and damping coefficient are calculated (Eq. 3.9 to 3.12).

Based on this method a computer program was developed [19] which performs the above calculations and was used in this investigation for the computation of the initial conditions of those motion parameters. Listings of those programs are presented in Appendix 2.

Still another way of calculating hydrodynamic coefficients is to use approximate formulas, based some of them on the same principles as exposed above,

and some in model experiments and full scale information. Some of those formulas are presented by Blagoveshchensky [20] and were used here to calculate A_{33} , A_{44} , B_{33} and B_{44} and the results compared with those obtained with the computer program mentioned, believed to be more exact. Those results were also compared with the ones obtained by Hines and Gies [21] in experiments with another model of the same hull form. Those results are presented in Appendix 3.

3.3 - Non-linear Hydrodynamic Coefficients

In this category are the terms of second order in velocity and cross terms in velocity and position.

Information on those types of parameters is practically nonexistent. Only two references to frictional damping in roll were found [20, 22]. Since that is the case where frictional and separation resistance has a bigger effect when compared with the other modes of motion, the expressions suggested by Blagoveshchensky was used since it was directly applicable to ocean vehicles.

The expression given by reference [22] relates the friction coefficient of oscillating cylinders to an equivalent Reynolds number:

$$c_f = 1.328 \left[\frac{3.22 r^2 \theta \dot{m}^2}{T \nu} \right]^{-.5} \quad (3.19)$$

where r - radius of the cylinder

θ_m - mean amplitude of the oscillation

T - period of oscillation

ν - kinematic viscosity

This formula is the same as Blasius solution for C_f in laminar flow if the Reynolds number is taken as

$$Re_y = \frac{3.22 r^2 \theta_m^2}{T \nu} \quad (3.20)$$

To obtain the drag coefficient for heaving oscillations a similar method may be used, although with rougher approximations since the vehicle is not a two dimensional body and the sections do not resemble circles.

Here a "strip theory" approach was used. Results were sought for each station and integrated over the length of the body.

For each station it is assumed that the resistance is due, when moving into the water, entirely to skin friction and when moving away from the water, entirely to separation. Therefore for the first case the diameter of the equivalent cylinder will be $d = l/\pi$ where l is the length of the underwater part of the contour of the section. For the second case the diameter of the equivalent cylinder will be equal to the beam of the section. The velocity used to calculate the

Reynolds number will be the "root mean square" value based on the tests performed. Values of skin-friction coefficient and form-drag coefficients are obtained from reference [23]. Results of this procedure together with the results of the coefficients as calculated by the method indicated in reference [20] are presented in Appendix 3.

With respect to the other coefficients, the assumption may be made that they are indeed very small and consequently the assigned value to be used in the identification procedure is zero.

Summarizing in this chapter the ship motion parameters were considered, their nature, importance and the available methods of prediction outlined. The results obtained for the chosen hull form are presented in Appendix 1, 2 and 3.

In the next chapter the exciting force and moment will be considered thus completing the description of the mathematical model.

CHAPTER 4

EXCITATION FROM THE ENVIRONMENT

From all possible types of excitation, only forces and moments due to the action of two dimensional plane progressive waves are considered because (1) they are well defined in mathematical terms and (2) easy to generate in a towing tank experiment.

The expression for the velocity potential is obtained following a path similar to the one followed in Chapter 3 for the calculation of relations between the linear hydrodynamic coefficients, namely that the velocity potential satisfies both Laplace and Bernoulli equations. Taking the case of an infinite fluid in all directions the following expressions hold [15, 23]:

$$\nabla^2 \bar{\Phi}_w = 0$$

$$\begin{aligned} \text{B.C.: } \left. \frac{d(\xi - x_3)}{dt} = 0 \right|_{x_3 = \xi} & \quad \left. \frac{\xi}{\delta t} \approx \frac{\xi w}{\delta x_3} \right|_{x_3 \approx 0} \\ p - p_{a+m} = 0 = - \left. \left(\frac{\xi \bar{\Phi}_w}{\delta t} + \frac{1}{2} (\nabla \bar{\Phi}_w)^2 - g \xi \right) \right|_{x_3 = \xi} & \quad (4.1) \\ \approx \left. \frac{1}{g} \frac{\xi \bar{\Phi}_w}{\delta t} \right|_{x_3 = 0} \end{aligned}$$

$$\nabla \bar{\phi}_w = 0 \quad x_3 \rightarrow \infty \quad (4.1)$$

$$\bar{\phi}_w = \text{finite as } r = \sqrt{x_1^2 + x_2^2} \rightarrow \infty$$

where \mathcal{S} is the surface displacement.

The solution is sought by separation of variables and assuming a harmonic dependence on time, or in other words, and for two dimensional behaviour:

$$w(x_1, x_2, x_3, t) = \text{Re}[\varphi_2(x_2) \cdot \varphi_3(x_3) \cdot e^{-i\omega t}] \quad (4.2)$$

Plugging this solution back into Laplace's equation we obtain

$$\frac{\varphi_2''(x_2)}{\varphi_2(x_2)} = -\frac{\varphi_3''(x_3)}{\varphi_3(x_3)} = -k^2 \quad (4.3)$$

To satisfy the last boundary condition k^2 must be positive being the solution for $\varphi_2(x_2)$ and $\varphi_3(x_3)$:

$$\varphi_2(x_2) = C_1 e^{ikx_2} + C_2 e^{-ikx_2}$$

$$\varphi_3(x_3) = D_1 e^{kx_3} + D_2 e^{-kx_3} \quad (4.4)$$

and to satisfy $\nabla \bar{\phi}_w = 0 \quad x_3 \rightarrow \infty \quad D_1 = 0$

$$\varphi_3(x_3) = D_2 e^{-kx_3}$$

and therefore

$$\bar{\phi}_w(x_2, x_3, t) = \text{Re} \left[e^{-kx_3} (C_1 e^{i(kx_2 - \omega t)} + C_2 e^{-i(kx_2 + \omega t)}) \right] \quad (4.5)$$

The solution with $C_2 = 0$ corresponds to waves progressing in the positive x_2 direction and with $C_1 = 0$ to waves progressing in the negative x_2 direction.

Assuming the later case and the existence of an arbitrary phase angle δ to be determined later from the initial conditions:

$$\Phi_w(x_2, x_3, t) = \text{Re } C e^{-[kx_3 + i(kx_2 + \omega t + \delta)]} \quad (4.6)$$

Combining the two boundary conditions on the surface ($x_3 \approx 0$)

$$\left. \begin{aligned} \frac{1}{g} \frac{\delta^2 \Phi_w}{\delta t^2} &= \frac{\delta w}{\delta x_3} \\ \frac{1}{g} \omega^2 &= k \end{aligned} \right|_{x_3 = 0} \quad (4.7)$$

which determines the value of k , the wave number.

$$\text{Using } \eta = \frac{1}{g} \frac{\delta \Phi_w}{\delta t} \quad \left|_{x_3 = 0} \quad 4.1\right.$$

$$\begin{aligned} &= \frac{1}{g} \text{Re} \left[-i\omega C e^{-i(kx_2 + \omega t + \delta)} \right] \\ &= \frac{\omega C}{g} \sin(kx_2 + \omega t + \delta) \end{aligned}$$

and for the form

$$\eta = \eta_a \sin(kx_2 + \omega t + \delta) \quad (4.8)$$

where η_a is the amplitude of the surface displacement

$$C = \frac{g\eta_a}{\omega} \quad (4.9)$$

and

$$\Phi_w(x_2, x_3, t) = \text{Re} \left[\frac{p_2}{\omega} e^{-[kx_3 + i(kx_2 + \omega t + \delta)]} \right] \quad (4.10)$$

from the Haskind relations, Eq. (3.16)

$$F_{wj} = \text{Re} \left[\frac{p}{s_{ja}} e^{-i\omega t} \iint_{S_0} \left[\varphi_w \frac{\delta \varphi_j}{\delta n} - \varphi_j \frac{\delta \varphi_w}{\delta n} \right] ds \right]$$

where $\varphi_w(x_1, x_2, x_3) = \Phi_w(x_1, x_2, x_3, t) e^{i\omega t}$

The first term in the integral represents the force exerted by the pressure in the wave without taking account of the disturbance by the presence of the ship and is the only excitation accounted for by the Froude-Kriloff hypothesis. The second term is the contribution of the diffraction of the waves from the body.

To compute the first term in the exciting force (the Froude-Kriloff force) first replace

$$\frac{\delta \varphi_j}{\delta n} = -i\omega f_j s_{ja} \quad (3.6)$$

$$\begin{aligned} (F_{wj})_{fk} &= \text{Re} \left[-i\omega p \iint_{S_0} \frac{s_{ja}}{\omega} e^{-[kx_3 + i(kx_2 + t + \delta)]} f_j ds \right] \quad (4.11) \\ &= -\rho g s_{ja} \iint_{S_0} e^{-kx_3} \sin(kx_2 + \omega t + \delta) f_j ds \end{aligned}$$

The second term (diffraction force) is computed by noting that

$$\frac{\delta \varphi_w}{\delta n} = -\omega f_2 e^{-[kx_3 + i(kx_2 + \delta)]} (f_2 + if_3) \quad (4.12)$$

$$(F_{wj})_d = \text{Re} \left[\frac{\rho \omega}{s_{ja}} e^{-i\omega t} \iint_{S_0} \varphi_j e^{-[kx_3 + i(kx_2 + \delta)]} (f_2 + if_3) ds \right] \quad (4.13)$$

and

$$F_{wj} = (F_{wj})_{fk} + (F_{wj})_d \quad (4.14)$$

Therefore to calculate the total excitation force the results from the sub-problem of forced oscillations of the body in calm water must be known, namely $\phi_j(x_1, x_2, x_3)$. What was said about the way of calculating added mass and damping coefficients is applicable here. Besides it is possible to calculate this second term in the exciting force at the same time as the added mass and damping coefficients are calculated, since the potential is also obtained in the mode of interest. Or, in other words, we obtain for each station

$$[(F_{wj})_d]_{2dim.} = \operatorname{Re} \left[\frac{\rho \omega}{s_{ja}} e^{-i\omega t} \int_{C_0} j e^{-[kx_3 + i(kx_2 + \delta)]} (f_2 + if_3) dl \right] \quad (4.15)$$

where C_0 is the contour of the station and dl is the length element along that contour.

These two dimensional results for each station are then integrated over the length of the vehicle.

Although this method seems to be the most suited for the computation of the exciting forces, that was not used in this investigation due to the fact that the computation of equation (4.15) was not included in the computer scheme used in the calculation of sectional added mass and damping coefficients. Instead, the

procedure presented by Blagoveshchensky [20] was used since, although less correct, rendered the computations much easier to perform.

First it is assumed that $\frac{\delta a}{\lambda} \ll 1$, $B/\lambda \ll 1$, $T/\lambda \ll 1$

where δa is the wave amplitude, λ is the wave length $\lambda = 2\pi/k$, B and T are the beam and the draft of the vehicle.

Next, it is assumed that the principal part of the excitation is due to change in buoyancy due to the presence of the wave. Then some multiplicative correction coefficients are calculated to account for the curvature of wave (finite beam correction) and for the dependence of the hydrodynamic pressure on x_3 (finite draft correction). Each correction is calculated separately and assumed independent from each other (for example, when calculating the beam correction, $T/\lambda \ll 1$ and vice versa).

Under these assumptions the linear equations for heave and roll will take the form:

a) heave

$$\dot{w} = \frac{Z_w}{-Z'_w + m} \cdot w + \frac{Z_{w_0}}{-Z'_w + m} (z_0 - \xi_c) \quad (4.16)$$

$$\dot{z}_0 = w$$

where ξ_c is the surface displacement corrected for finite beam and draft effects

$$\begin{aligned}\xi_c &= \xi \cdot C_{zb} \cdot C_{zt} \\ &= a \sin(ky + t + \delta)\end{aligned}$$

b) roll

$$\dot{p} = \frac{K_p}{-K_p + I_{xx}} r + \frac{[K_\phi - W.BG](\phi - \xi_y)}{-K_p + I_{xx}} + \frac{W.BG}{-K_p + I_{xx}} \phi \quad (4.17)$$

$$\dot{\phi} = p$$

where ξ_y is the wave slope corrected for finite beam and draft effects

$$\xi_y = \xi_y \cdot C_{\phi b} \cdot C_{\phi t}$$

$$\xi_y = \frac{\partial \xi}{\partial y} = k \xi_a \cos(ky + \omega t + \delta)$$

C_{zb} , C_{zt} , $C_{\phi b}$, $C_{\phi t}$ are the multiplicative corrections given by [20]

$$C_{zb} = 1 - 1.73 C_w (B/\lambda)^2$$

$$C_{zt} = 1 - C_m(kT) + \frac{C_m}{2(2-C_m)} (kT)^2 - \frac{C_m}{6(3-2C_m)} (kT)^3 \quad (4.18)$$

$$C_{\phi b} = 1 - C_w (B/\lambda)^2$$

$$\begin{aligned}C_{\phi t} &= 1 - \frac{6 C_m^3}{(1+C_m)(1+2C_m)} (kT) + \frac{1.5 C_m^3}{(2-C_m)(2+C_m)} (kT)^2 - \\ &\quad \frac{C_m^3}{3(3-2C_m)(3-C_m)} (kT)^3\end{aligned}$$

where

C_w is the waterline coefficient $C_w = A_{WL}/BL$

C_m is the midship section coefficient $C_m = A_{ST}/BT$

k is the wave number.

The details of the calculation of these correction coefficients may be found in Reference [20] paragraphs 27 to 31.

In Equation 4.17 the term $\frac{W \cdot BG}{-k_p + I_{xx}}$ represents

the contribution due to the inclination of the ship when $\phi = \xi_z$ since the weight of the vehicle and its buoyancy will not be in the same vertical thus causing a heeling moment $W \cdot BG \sin \phi$ being $W \cdot BG \cdot \phi$ the first term in the Taylor's expansion. W is the weight of the vehicle and BG is the distance from the center of gravity to the center of buoyancy.

Although the expressions above for the exciting force and moment are not as correct as Equations 4.11, 4.13 and 4.14, the simplicity of their calculation decided their introduction in the equations of motion.

When considering the non-linearities, if the exciting force can still be approximated as expressed in Equation 4.16 and 4.17, the following terms should be added to each pair of equations.

a) Heave

$$\frac{1/2 Z_{wz}}{-Z_w + m} \cdot w|w| \quad \text{and} \quad \frac{1/2 Z_{z_0 z}}{-Z_w + m} \cdot (z_0 - \xi_z)^2 \quad (4,19)$$

where the first term represents the frictional and form drag and the other term is the second term in the

polynomial fit of the $Z(z_0, \phi) \Big|_{\phi=0}$

b) roll

$$\frac{1/2 k_{p2}}{-k_p + I_{xx}} \cdot p|p|, \quad \frac{1/3! (k_{\phi^3} + W.BG)}{-k_p + I_{xx}} (\phi - \xi_2)^3 \text{ and} \quad (4.20)$$

$$\frac{1/3! W.BG}{-k_p + I_{xx}} \cdot \phi^3$$

Where the first term is again the contribution of frictional and form drag, the second term is the contribution from the polynomial fit of the curves of stability and the last is the following term in the expansion of $GB \sin \phi$.

In the case above, the validity of the expression for the excitation is even more questionable due to the small orders of magnitude involved in the identification. One way of obtaining a better approximation is to consider the effects of the presence of the wave in the damping terms as is done for the case of roll by Tanya [22].

Those terms would then look like

a) heave

$$\frac{Z_w}{-Z_w + m} \cdot (w - \dot{\xi}_c) \quad \text{and} \quad \frac{1/2 Z_{w2}}{-Z_w + m} \cdot (w - \dot{\xi}_c) \cdot |w - \dot{\xi}_c| \quad (4.21)$$

b) roll

$$\frac{k_p}{-k_p + I_{xx}} \cdot (p - \dot{\varphi}_2) \quad \text{and} \quad \frac{1/2 k_{p2}}{-k_p + I_{xx}} \cdot (r - \dot{\varphi}_2) | r - \dot{\varphi}_2 | \quad (4.22)$$

where

$$\dot{\varphi}_2 = \frac{\delta \varphi_2}{\delta t}$$

$$\dot{\varphi}_2 = \frac{\delta \varphi_2}{\delta t}$$

When considering the non-linear coupled heave and roll mode, all the coupling terms as mentioned in Chapter 3 had to be added from which only the pure hydrostatic terms could be predicted by using, for example, the method presented in References [6, 7]. The same objections would apply with respect to the excitation.

In summary, the excitation due to the action of plane progressive waves can be calculated quite accurately using the Haskind relations, being the action from the diffraction potential calculated for each station by two dimensional techniques and the resultant force and moment obtained by integration of those two dimensional results along the length of the vehicle. Due to computational difficulties another method was used which gives results believed to be less accurate but offers the advantage of its simplicity. Another advantage not mentioned before is that this later method express

the exciting forces and moments in terms of instantaneous values of surface displacement and wave slope which are easy to obtain during the tank experiments setup to measure displacements and velocities that will be used in the identification procedure.

CHAPTER 5

EXPERIMENTAL PROCEDURE

A model of the "mariner" type hull form, whose characteristics are given in Table 5.1, was tested in regular waves measuring the displacements and velocities in heave, roll and coupled heave and roll, and also the surface elevation, with the purpose of identifying the hydrodynamic coefficients in those modes of motion.

TABLE 5.1 - MODEL CHARACTERISTICS

scale (model/full size)	1/96
length (L)	5.50 ft.
beam (B)	.79 ft.
draft (T)	.28 ft.
weight (W_0)	46.05 lb.
mass (m)	1.43 lb.sec ² /ft.

The model was placed in the mid-length of the tank with its centerline perpendicular to the centerline of the tank such that, using the coordinate system as defined in Chapter 2, the direction of propagation of the waves generated at the wavemaker in the beginning of the tank, coincide with the negative $x_2 = y$ direction.

The model was placed at the mid-length of the tank

as a solution of compromise: not too close to the wavemaker in order to avoid the action of the diffracted waves from the model upon reflexion on the wavemaker, not too far from the wavemaker in order to avoid interference from the walls of the tank. Reflexion on the opposite end of the wavemaker was of much less concern due to the existence of a "beach" composed of stainless steel shavings with a reflexion coefficient of about .05, for normal wave lengths.

Tests were run at three different frequencies. First it was decided that those frequencies would be half, twice and the natural undamped frequency for both heave and roll, where the natural undamped frequency is defined as for a linear system

$$\omega_n = \sqrt{k/m} \quad \text{where} \quad k = -Z_{z0} = -k_\phi$$

$$m = -Z_w + m = -K_p + I_{xx}$$

For roll the measured natural period (from free oscillations of the model in calm water) was $T_n = 1.52$ seconds or $f_n = .66$ cps. Therefore tests were run at

$$\begin{aligned} f_1 &= .33 \text{ cps} & \lambda &= gT^2/2\pi &= 47.36 \text{ ft.} \\ f_2 &= .66 \text{ cps} & & &= 11.84 \text{ ft.} \\ f_3 &= 1.32 \text{ cps} & & &= 2.96 \text{ ft.} \end{aligned}$$

For heave the measured natural period was $T_n = .78$ seconds or $f_n = 1.28$ cps. For the tests to be run at twice the natural frequency $f = 2.48$ cps, it would be too close to the maximum at 2.50 cps permitted by the wave maker. Therefore, it was decided to run the tests in heave at one fourth, half and at the natural frequency, or

$f_1 = .32$ cps	$\lambda = 49.92$ ft.
$f_2 = .64$ cps	$= 12.48$ ft.
$f_3 = 1.28$ cps	$= 3.12$ ft.

For the tests in coupled heave and roll, it was decided to run at the natural frequency in heave and at half and at the natural frequency in roll, therefore, for the coupled mode

$f_1 = .33$ cps	$\lambda_1 = 47.36$ ft.
$f_2 = .66$ cps	$\lambda_2 = 11.84$ ft.
$f_3 = 1.28$ cps	$\lambda_3 = 3.12$ ft.

The quantities measured in each test were
a) heave

surface displacement $\xi(t)$
heave displacement $z_0(t)$
heave acceleration $\ddot{z}_0(t)$

The reason for measuring acceleration instead of velocity was due to the impossibility of obtaining a translational velocity transducer, and to not being able to build a reliable electronic integrator with the material available

b) roll

surface displacement	$\xi(t)$
roll displacement	$\phi(t)$
roll velocity	$p(t)$

Again wave slope should be measured in this mode instead of surface elevation but the probe used could only measure $\xi(t)$ ¹

c) coupled heave and roll

surface displacement	$\xi(t)$
heave displacement	$z_o(t)$
heave acceleration	$\ddot{z}_o(t)$
roll displacement	$\phi(t)$
roll velocity	$p(t)$

The same observations as made in a) and b) above apply.

¹ Description of probes with provision to measure simultaneously surface displacement and wave slope is available in Reference [24].

Surface displacement was measured with resistance type wave probe formed by two parallel stainless steel wires of 1/8" diameter. Since it was necessary to obtain the wave displacement at the vehicle, the ideal position for the wave probe would be along the centerline of the model. Since at this position the probe would be subject to interference between the model and the tank walls, it was decided to put the probe in integer number of wave lengths ahead of the model. Those distances were calculated based on the relation $k = \omega^2/g$ or $\lambda = gT^2/2\pi$ and this procedure introduced the first error on the measurements since for finite water depths the relation between wave number and frequency is [23].

$$\frac{\omega^2}{g} = k \tanh(kh) \quad (5.1)$$

where h is the water depth.

For example, considering $h = 4$ ft.

$f = \omega/2\pi$	$(\lambda_\infty - \lambda_h)/\lambda_\infty$
.5 cps	.098
.65 cps	.025
7.85 cps	≈ 0

From the above it is seen that for the low frequencies $f < .5$ cps the error becomes appreciable.

Heave displacement was measured with a linear variable differential transformer (LVDT) connected to the heave rod assembly. The heave rod slides on a sleeve provided with ball bearings to reduce friction. Even though the friction was found to be quite high (comparing with experiments had with a heave rod assembly provided with air bearings) and therefore there will be some error involved in the identification of the heave damping parameters.

Heave acceleration was measured with an accelerometer placed in the model as close as possible to the axis of rotation in order to minimize interference of roll in the measurement of heave acceleration.

Roll displacement was measured by a rotational variable differential transformer attached to a roll bearing.

Roll velocity was measured by means of a rate gyro, borrowed from the Charles Draper Laboratory, which was placed in the model with its three axes of rotation parallel to the coordinate axes located on the model. This rate gyro, used in the Deep Submergence Systems Project, was able of measuring angular rates in roll, pitch and yaw simultaneously, up to rates of 40 degrees/second.

All the transducers mentioned above, with the exception of the rate gyro, are excited by an A.C. voltage of 5 volts at a frequency of 2400 cps provided by carrier preamplifiers Sanborn model 8805A which also received the electrical response from the transducers, generating a D.C. signal which may be fed to a chart or tape recorder or both. The amplifiers also provide means of amplifying or attenuating the electrical signal according to its level since both recorders work between ± 2.5 volts.

Since the excitation necessary for the rate gyro is 26 V A.C. ($f = 400$ cps), the Sanborn preamplifiers could not be used. Instead the following circuit was used (see Fig. 5.1), since a D.C. power supply was available up to three amps:

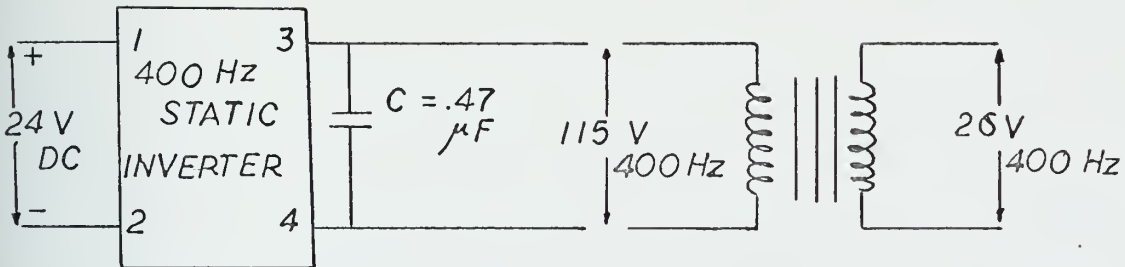


Figure 5.1

The 26 V 400 hz excitation was fed to the rate gyro. The output from the rate gyro (400 hz, phase

reversing) was then fed to a demodulator (PRD type 808) to obtain the D.C. signal proportional to the angular velocity. This signal was attenuated in a Sanborn D.C. amplifier before going into the chart and tape recorder (see Fig. 5.2).

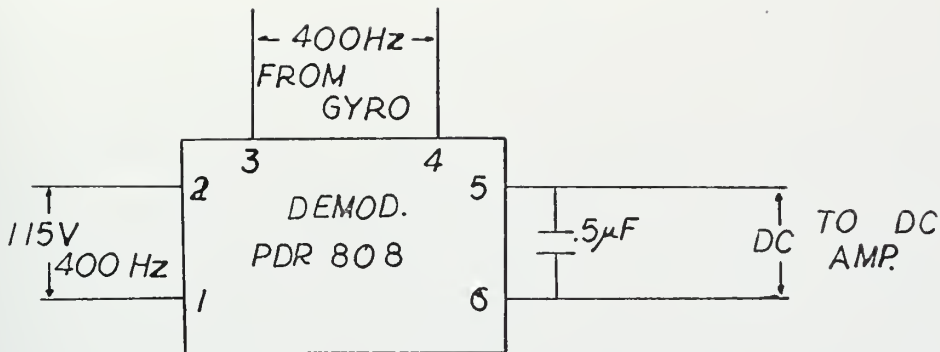


Figure 5.2

For convenience of future calculation, it was decided to put the center of gravity at the design waterline (all stability calculations were made for an assumed KG = .28 ft. - center of gravity at the waterline). This was performed by trial and error by creating a known moment (with the model in the upright position, a given weight is moved transversally a certain given distance) and measuring the resulting inclination which was compared

with the value calculated in the curves of stability. Weights were then moved in the vertical and the angle verified again until reaching the value specified in the curves.

Calibration of all instrumentation was made in the tank except for the rate gyro which was performed at the Draper Laboratory.

The LVDT was first zeroed by sliding the core into the casing until no output was shown on the chart recorder. This point was marked on the shaft of the core together with marks spaced 1" on each side. Then for a movement of the core of 1" and for a given attenuation of the pre-amplifiers the per deflection on the chart recorder was noted (1" = 10 mm - 50 mv/mm attenuation factor). For the wave probe and rotational differential transformer the procedure was similar. For the rotational transformer using a protractor designed for that purpose. The calibration of the heave rod and roll bearing was checked later with the instrumentation aboard.

For the calibration of the accelerometer zero was checked with it in the horizontal. The accelerometer was then turned 90 degrees and its full scale value (1.5 g \leftrightarrow 48.3 ft./sec.²) given full scale deflection of

the pen (25 mm). Then the zero suppression control on the preamplifier was adjusted to bring the output to zero.

For the rate gyro, the whole assembly (rate gyro, inverter, transformer and demodulator) was placed in a rotating table to which a known constant angular velocity was given. The output voltage was then measured with a high precision D.C. voltmeter. It was noted that the presence of demodulator introduced some hysteresis on the output as well as some non-linearity above an angular velocity of 20 degrees/second. On the average the output signal was of .136V/degree/second D.C. while for the rate gyro alone it was .147V/degree/second rms.

Afterwards in the tank a constant angular velocity was simulated by means of a potentiometer, the output voltage read with an oscilloscope and the pen deflection on the chart recorder noted.

The results of the final calibration for all the instruments were as follows:

wave probe	1" = 10 mm (atten. factor = 10)
heave rod	1" = 10 mm (atten. factor = 50)
accelerometer	1.5 g = 25 mm (atten. factor = 50)
roll bearing	10° = 10 mm (atten. factor = 200)
rate gyro	20°/sec. = 10 mm (atten. factor = 200)

From the above some other mistakes can be noted.

First, full scale (25 mm) corresponds to 2.5 volts. For a model of 3.375 inches of draft heave displacements of more than 1 inch ($1/3 T$) can hardly be expected. Therefore, the output signal on tape will vary between ± 1 volt at most. For the acceleration if it is assumed that, as for linear systems $\ddot{z}_a = \omega^2 z_a$ the amplitude of the acceleration would be for the highest frequency ($f = 1.28$ cps

$= 8.04 \text{ sec.}^{-1}$) $z_a = 64.64 \text{ inch/sec.}^2$ or 5.38 ft./sec.^2 since full scale was $1.5 g = 48.3 \text{ ft./sec.}^2$ the signal on tape would vary between $\pm .25$ volts. For the lower frequencies the acceleration output was hardly perceptible on chart paper and not usable for the digitization, since output signal approached the noise level.

Similarly for roll displacement and velocity and for the wave probe, the signal level was quite low.

Before running the tests calibration signals were sent into the tape recorder in order to be able to scale the data to correct values of displacements and velocities once the digitization being performed. About 50 ft. of tape were recorded with those calibration signals at a recorder speed of $7\frac{1}{2}$ inch/sec.

Then the nine tests as mentioned before were run

recording about 50 cycles for each run. In between runs enough time would elapse in order for the model and the water to calm down (zero initial conditions). On tape the interval between runs was about 20 ft.

The waves were generated by a wave maker constituted by a hinged paddle actuated by an electro-hydraulic power system, which was controlled by electrical sinusoidal signals generated by a low frequency wave oscillator, through a hydraulic servo valve.

The data was digitized from the magnetic tape into computer cards by means of the EAI 680 analog computer, EAI 693 data conversion system and the 1130 IBM digital computer of the Department of Mechanical Engineering. During digitization some more deficiencies were noted, besides the low level of the signals on tape (the EAI 680 works within a voltage level of ± 10.5 volts in normal operation; when digitized these signals correspond to a standard precision real value of ± 1.05). First the magnetic tape had, on the portions not recorded, a D.C. signal of about 5.8 volts whose origin couldn't be determined. A signal of this magnitude could magnetize the heads of the tape recorder when on "play", thus introducing errors on the values of the measurements.

Also the runs were too short to be possible to set up the values of the potentiometers conveniently. These potentiometers were used to take off any bias that existed in the measurements.

For purposes of digitization, these circuits were built in the EAI 680 analog computer (refer to Appendix 4 and Reference [25] for details in the circuitry):

1. - Control of Sampling Rate

A Wavetek 116 VCO oscillator is used to generate a square wave of frequency equal to the sampling rate. This signal is obtained in analog trunks 0 or 3. From the trunks the signal goes into the input of a comparator in the interface tray of the analog rows. The output, which is obtained in the same interface tray in the logic row is fed into a differentiator to obtain a spike. The output of the differentiator is in turn fed into the analog to digital converter (ADC) sample control terminals on the first strip, preferably on trunk "all". This signal may also be fed to a BDC counter in Column 01 to check the operation of the circuit.

2. - Signal Processing

From the tape recorder the signals are fed into analog trunks 14 to 20, located below the Wavetek

oscillator. From the trunks whose output is situated in the second strip the signal goes into a summer together with the output of a potentiometer (to take off any bias on the signal). The output of the summer is fed into a potentiometer and from the potentiometer into an amplifier (input marked with a 10). The purpose of the potentiometer is to attenuate, if necessary, the signal which is amplified ten times in the amplifier. The signal correspondent to the measured acceleration, instead of going into an amplifier, is fed into an integrator to convert it to velocity. The output of the amplifier or integrator is then fed into the ADC trunks (located in the second strip) to be digitized and at the same time into the display trunks for display into the GPS DU300T oscilloscope.

3. - Display Circuit

From terminal B for example, in the mode control timer (in column 02 of the logic row) a wire is connected to the "scope" trunks - DIS terminal located TRC in the first strip in order to obtain a display in any analog mode control. A pin is introduced in the "run" terminal located close to terminal B mentioned above.

Digitization was initiated in the IBM 1130 by a computer program built according to the hybrid user's guide published by the Department of Mechanical Engineering using existing routines. This computer program is presented in Appendix 4.

The output from the digitization was punched into computer cards.

The selected sampling rate was twenty samples/cycle for fifty cycles for all runs. Such a frequent sample will allow a method of linear interpolation to be used in the identification of the parameters.

In summary the main points in the experimental procedure were outlined. The mistakes made, due mainly to inexperience in this kind of experiments, were also pointed out. If time permitted they should be repeated in order to obtain better data.

Digitization was explained in some extent in order to allow a somewhat easier handling of data in the future.

CHAPTER 6

IDENTIFICATION OF SHIP MOTION MODEL PARAMETERS

Most of the available techniques of parameter identification have been developed for linear dynamic systems [1]. Some of those techniques, assuming the linearized equations of motion to be a valid description of the behaviour of an ocean vehicle, have already been applied in the determination of ship motion parameters in heave and pitch namely, the frequency response [27] and the step response methods [28].

In the case of a non-linear model, two approaches are open, both yielding approximate results.

The first, making no assumptions about the characteristic of the uncertainty, assumes a mathematical model for the system and then comparing the output of both model and systems to the same input, searches for the structure of the model that minimizes a function of the error between the two outputs. An example of the foregoing is the model reference technique. For the system:

$$\begin{aligned} \underline{y}_s(t) &= \underline{f}_s(\underline{u}(t_o, t), t) \\ \underline{y}_m(t) &= \underline{f}_m(\underline{u}(t_o, t), t) \\ V &= F(\underline{y}_s - \underline{y}_m) \end{aligned} \tag{6.1}$$

where \underline{u} vector input to both system and model

\underline{y}_s - vector output of the system

\underline{y}_m - vector output of the model

\underline{f}_s - vector structure of the system

\underline{f}_m - vector structure of the model

V - scalar function to be minimized.

A simplified version of the above will be

$$\begin{aligned}\underline{y}_s(t) &= \underline{f}(\underline{u}(t_0, t), t, \underline{p}_s) \\ \underline{y}_m(t) &= \underline{f}(\underline{u}(t_0, t), t, \underline{p}_m)\end{aligned}\quad (6.2)$$

where the values of \underline{p}_s are unknown and the values of \underline{p}_m will be determined by minimizing V , which is often of the form

$$V(\underline{p}_m, t_0, t) = \frac{1}{t-t_0} \int_{t_0}^t [\underline{y}_s(\tau) - \underline{y}_m(\tau)] [\underline{y}_s(\tau) - \underline{y}_m(\tau)]^T d\tau \quad (6.3)$$

and for optimum \underline{p}_m (best approximation) $\frac{\delta V}{\delta \underline{p}_m} = 0$. The

vector \underline{p} thus found will only be optimum in the interval $t_0 < \tau < t$ and may not apply outside that interval. Also it may not be true for non-linear systems that

$$\lim_{t \rightarrow \infty} \underline{p}_m = \underline{p}_s \quad (6.4)$$

as one might expect from linear systems. The optimum value of \underline{p}_m in Equation (6.3) could be found by a maximum gradient seeking algorithm (first order gradient method) or by a second order gradient method [29]

The second approach is to use linear estimation techniques, such as a Kalman filter to the linearized form of a modified state representation of the system.

Considering a linear system driven by noise, Bryson and Ho [29, Chp. 11 and 12] present the following expressions for the estimate of the system:

1) Linear Multistage Systems

$$\underline{x}_{i+1} = \underline{F}_i \underline{x}_i + \underline{G}_i \underline{w}_i$$

$$\begin{aligned} \underline{x} - \text{state vector} \quad E[\underline{x}_0] &= \bar{\underline{x}}_0 \\ E[\underline{x}_0 - \bar{\underline{x}}_0] [\underline{x}_0 - \bar{\underline{x}}_0]^T &= M_0 \end{aligned} \quad (6.5)$$

$$\underline{w} - \text{environment noise} \quad E[\underline{w}] = \bar{\underline{w}}$$

$$E[\underline{w}_i - \bar{\underline{w}}_i] [\underline{w}_j - \bar{\underline{w}}_j]^T = Q_i \delta_{ij}$$

$$E[\underline{w}_i - \bar{\underline{w}}_i] [\underline{x}_0 - \bar{\underline{x}}_0]^T = 0$$

$\underline{F}_i \underline{G}_i$ - transition matrices

$$\text{and } \underline{z}_i = \underline{H}_i \underline{x}_i + \underline{v}_i$$

\underline{z} - measurement vector

$$\underline{H}_i - \text{measurement matrix} \quad (6.6)$$

$$\underline{v}_i - \text{measurement noise} \quad E[\underline{v}_i] = 0$$

$$E[\underline{v}_i - \bar{\underline{v}}_i] [\underline{v}_j - \bar{\underline{v}}_j]^T = R \delta_{ij}$$

$$E[\underline{x}_0 - \bar{\underline{x}}_0] \underline{v}^T = 0$$

Assuming the sequences i of the different vectors to represent Gauss-Markov random sequences, they present the solution for the estimate of the state $\underline{x} \longrightarrow \hat{\underline{x}}$ as:

$$\hat{\underline{x}}_i = \bar{\underline{x}}_i + \underline{K}_i (\underline{z}_i - \underline{H}_i \bar{\underline{x}}_i)$$

$$\bar{\underline{x}}_{i+1} = \underline{F}_i \hat{\underline{x}}_i + \underline{G}_i \underline{w}_i$$

$$\underline{K}_i = \underline{P}_i \underline{H}_i^T \underline{R}_i^{-1} \quad (6.7)$$

$$\underline{P}_i = [\underline{M}_i^{-1} + \underline{H}_i^T \underline{R}_i^{-1} \underline{H}_i]^{-1} =$$

$$\underline{M}_i - \underline{M}_i \underline{H}_i^T (\underline{H}_i \underline{M}_i \underline{H}_i^T + \underline{R}_i)^{-1} \underline{H}_i \underline{M}_i$$

$$\underline{M}_{i+1} = \underline{F}_i \underline{P}_i \underline{F}_i^T + \underline{G}_i \underline{Q}_i \underline{G}_i^T$$

where \underline{P}_i is the error covariance matrix of \underline{x}_i after the measurement while \underline{M}_i is the error covariance matrix before the measurement. Similarly $\hat{\underline{x}}_i$ is the estimate of the system after the measurement while $\bar{\underline{x}}_i$ is the estimate of the system before measurement.

2) Linear Continuous Systems with Continuous Measurements

The mathematical expression for this type of system is obtained by letting the time between stages go to zero. Therefore, for the system

$$\dot{\underline{x}} = \underline{F}\underline{x} + \underline{G}\underline{w} \quad (6.8)$$

$$\underline{z} = \underline{H}\underline{x} + \underline{v}$$

$$\text{with } \underline{F} = \lim_{\Delta \rightarrow 0} \frac{F(t_i + \Delta, t_i) - \underline{I}}{\Delta} \quad (6.8)$$

$$\underline{G} = \lim_{\Delta \rightarrow 0} \underline{G}_i$$

Under the assumption that \underline{x} represents a Gauss-Markov random process, the expression for the Kalman filter will be

$$\dot{\underline{\hat{x}}} = \underline{F}\underline{\hat{x}} + \underline{G}\underline{\bar{w}} + \underline{K}(\underline{z} - \underline{H}\underline{\hat{x}})$$

$$\underline{K} = \underline{P} \underline{H}^T \underline{R}^{-1}$$

$$\dot{\underline{P}} = \underline{F} \underline{P} + \underline{P} \underline{F}^T + \underline{G} \underline{Q} \underline{G}^T - \underline{P} \underline{H}^T \underline{R}^{-1} \underline{H} \underline{P}$$

$$\underline{P}(t_0) = \underline{P}_0$$

If, for the system as specified in (1.11) to (1.13)

$$\dot{\underline{x}}' = \underline{f}'(\underline{x}', \underline{u}, \underline{p}') + \underline{w}' \quad (1.11)$$

$$\dot{\underline{p}} = 0 \quad (1.12)$$

$$\underline{z} = \underline{H}' \underline{x}' + \underline{v} \quad (1.13)$$

the above technique is to be used, it is necessary by first to form the extended state vector

$$\underline{x} = \begin{bmatrix} \underline{x}' \\ \underline{p} \end{bmatrix} \quad (6.10)$$

The new state equations are as follows:

$$\dot{\underline{x}} = \underline{f}(\underline{x}, \underline{u}) + \underline{w} \quad E[\underline{x}(t_0)] = \underline{\bar{x}}_0 \quad (6.11)$$

$$\underline{z} = \underline{H} \underline{x} + \underline{v}$$

where in addition to 6.10

$$\underline{w} = \begin{bmatrix} \underline{w}' \\ 0 \end{bmatrix} \quad (6.12)$$

$$\underline{f} = \begin{bmatrix} \underline{f}' \\ \underline{0} \end{bmatrix} \quad (6.12)$$

$$\underline{H} = [\underline{H}' \quad \underline{0}]$$

Now the state equations are linearized in order to use Equation (6.9) or:

$$\begin{aligned} \dot{\underline{x}} &= \underline{f}(\underline{x}, \underline{u}) + \underline{\bar{w}} + \underline{K}(\underline{z} - \underline{H} \underline{x}); \underline{x}(t_0) = \underline{\bar{x}}_0 \\ \underline{K} &= \underline{P} \underline{H}^T \underline{R}^{-1} \\ \dot{\underline{P}} &= \frac{\delta \underline{f}}{\delta \underline{x}} \underline{P} + \underline{P} \frac{\delta \underline{f}}{\delta \underline{x}}^T + \underline{Q} - \underline{P} \underline{H}^T \underline{R}^{-1} \underline{H} \underline{P}; \underline{P}(t_0) = \underline{P}_0 \end{aligned} \quad (6.13)$$

The next simplification will be, considering that in practice most measurements are discrete in time, to approximate the above differential equations by a set of difference equations and use the formulation for a linear multistage processes or, in other words, for the system below :

$$\underline{x}_{i+1} = \underline{x}_i + \Delta \cdot \underline{f}(\underline{x}_i, \underline{u}_i) + \Delta \cdot \underline{w}_i \quad (6.14)$$

$$\underline{z}_i = \underline{H} \underline{x}_i + \underline{v}$$

The resulting filter will be, assuming for convenience that

$$\begin{aligned} E[\Delta \cdot \underline{w}_i] &= \underline{0} \\ \hat{\underline{x}}_{i+1} &= \underline{\bar{x}}_i + \Delta \cdot \underline{f}(\underline{x}_i, \underline{u}_i) + \Delta \cdot \underline{K}_{i+1}(\underline{z}_i - \underline{H} \underline{\bar{x}}_i) \end{aligned} \quad (6.15)$$

$$\underline{K}_i = \underline{P}_i \underline{H}^T (\underline{R} \cdot \Delta)^{-1}$$

$$\begin{aligned} \underline{P}_i &= \underline{M}_i - \Delta \cdot \underline{M}_i \underline{H}^T (\underline{H} \underline{M}_i \underline{H}^T \Delta + \underline{R} \cdot \Delta)^{-1} \underline{H} \underline{M}_i \\ \underline{M}_{i+1} &= \frac{\delta}{\delta \underline{x}_i} (\underline{x}_i + \Delta \cdot \underline{f}(\underline{x}_i, \underline{u}_i)) \underline{P}_i \frac{\delta}{\delta \underline{x}_i} (\underline{x}_i + \Delta \cdot \underline{f}(\underline{x}_i, \underline{u}_i))^T + \underline{Q} \cdot \Delta \end{aligned} \quad (6.15)$$

where \underline{R} and \underline{Q} are constant matrices. If the value of Δ is sufficiently small, this formulation will represent closely Equations (6.13) and will be much easier computationally.

In the above formulations, the partial derivatives may be evaluated in a nominal path or if greater accuracy is desired they are evaluated at $\underline{x} = \hat{\underline{x}}$.

Bryson and Ho [29] mention several authors who have achieved success using Equations (6.13) and (6.15) for non-linear systems but at the same time make the warning that convergence may not be obtained if the initial guess is poor or if the disturbances are so large that the linearization is inadequate to describe the system.

From the foregoing Equations (6.2) and (6.3), Equations (6.13) and Equations (6.15) it was decided to use Equations (6.15) due mainly to computational convenience. Due also to the poorness of the available data, as mentioned in Chapter 5, it was decided to abandon the identification in heave and coupled heave and roll

and concentrate exclusively on the identification of roll motion parameters whose data at a frequency of excitation of .66 cycles/second (roll natural undamped frequency) seemed reasonably good.

As mentioned in Chapter 4, Equation (4.17) the state equations for roll motion, ignoring for simplicity the dependency of the system on the derivative of the input are:

$$\begin{aligned} \dot{p} = & \frac{K_p}{-K_p + I_{xx}} p + \left[\frac{K_\phi}{-K_p + I_{xx}} - \frac{W.BG}{-K_p + I_{xx}} \right] (\phi - \phi_y) + \\ & \frac{W.BG}{-K_p + I_{xx}} + \left[\frac{1/2! K_p |p|}{-K_p + I_{xx}} \right] p |p| + \left[\frac{1/3! K^3}{-K_p + I_{xx}} + \right. \\ & \left. \frac{1/3! W.BG}{-K_p + I_{xx}} \right] (\phi - \phi_y)^3 + - \left[\frac{1/3! W.BG}{-K_p + I_{xx}} \right] \phi^3 \end{aligned} \quad (4.17)$$

$$\dot{\phi} = p$$

Non-dimensionalizing the above equations with the frequency of excitation the following is obtained:

$$\begin{aligned} \left[\frac{\dot{p}}{\omega^2} \right] = & \left[\frac{K_p}{-K_p + I_{xx}} \cdot \frac{1}{\omega} \right] \left[\frac{p}{\omega} \right] + \left[\frac{K}{-K_p + I_{xx}} \cdot \frac{1}{\omega^2} - \frac{W.BG}{-K_p + I_{xx}} \cdot \frac{1}{\omega^2} \right] \\ & [\phi - \phi_y] + \left[\frac{W.BG}{-K_p + I_{xx}} \cdot \frac{1}{\omega^2} \right] \phi + \left[\frac{1/2! K_p |p|}{-K_p + I_{xx}} \right] \frac{p}{\omega} \cdot \frac{|p|}{\omega} \end{aligned} \quad (6.16)$$

$$+ \left[\frac{1/3!K}{-K_p + I_{xx}} \frac{1}{\omega^2} + \left[\frac{1/3!W.BG}{-K_p + I_{xx}} \frac{1}{\omega^2} \right] (\phi \cdot \dot{\phi})^3 - \left[\frac{1/3!W.BG}{-K_p + I_{xx}} \frac{1}{\omega^2} \right] \phi^3 \right] \quad (6.16)$$

$$\begin{bmatrix} \dot{\phi} \\ \omega \end{bmatrix} = \begin{bmatrix} p \\ \omega \end{bmatrix}$$

Based on results of Appendices 1, 2 and 3, the following non-dimensional parameters will be, for

$$\omega = 4.145 \text{ sec.}^{-1};$$

$$\frac{K_p}{-K_p + I_{xx}} \frac{1}{\omega} = -.0042 = .01 \text{ x(3)}$$

$$\frac{K_p}{-K_p + I_{xx}} \frac{1}{\omega^2} = -.989 = \text{ x(4)}$$

$$\frac{W.BG}{-K_p + I_{xx}} \frac{1}{\omega^2} = 2.744 = 10 \text{ x(5)} \quad (6.17)$$

$$\frac{1/2!K_p p}{-K_p + I_{xx}} = -.126 = \text{ x(6)}$$

$$\frac{1/3!K^3}{-K_p + I_{xx}} \frac{1}{\omega^2} = 3.123 = 10 \text{ x(7)}$$

Also and to have the values of all the state variables between ± 1 , it was chosen that

$$\frac{\dot{p}}{\omega^2} = .1 \text{ y(1)}$$

$$\frac{p}{\omega} = .1 \text{ x(1)} \quad (6.18)$$

$$\frac{\dot{\phi}}{\omega} = .1 \text{ y(2)}$$

$$\begin{aligned} &= .1x(2) \\ &= .1u(1) \end{aligned} \quad (6.18)$$

The complete set will then have the form :

$$\begin{aligned} .1y(1) &= .01x(3).1x(1) + [x(4)-10x(5)] [.1x(2)-.1u(1)] + \\ &+ 10x(5).1x(2) + \\ &+ x(6).1x(1).1|x(1)| - [10x(7)-.167 10x(5)] \\ &[.1x(2)-.1u(1)]^3 - .167 10x(5)[.1x(2)]^3 \\ .1y(2) &= .1x(1) \end{aligned}$$

or, rearranging

$$\begin{aligned} y(1) &= .01 x(3) x(1) + [x(4) - 10 x(5)] [x(2) - u(1)] + \\ &+ 10 x(5) x(2) + \\ &+ .1 x(6) x(1) \text{ABS } x(1) + [.1 x(7) - .0167 x(5)] \\ &[x(2) - u(1)]^3 - .0167 x(5) x(2)^3 \\ y(2) &= x(1) \end{aligned} \quad (6.19)$$

Before using the data, this model was tested by simulating noisy measurements — only $x(1)$ and $x(2)$ were measurable in the simulation — and the control $u(1)$, with a format similar to the one obtained in the data collected.

Also simplified forms of the state equations were tested. Their expression were :

Non-linear equations of motion to Identify linear terms

$$\begin{aligned} y(1) &= .01 x(3) x(1) + [x(4) - 10 x(5)] [x(2) - u(1)] + \\ &+ 10 x(5) x(2) + .0126 x(1) \text{ABS}(x(1)) + \\ &+ [.03123 - .0167 x(5)] [x(2) - u(1)]^3 + \\ &+ .0167 x(5) x(2)^3 \end{aligned} \quad (6.20)$$

$$y(2) = x(1) \quad (6.20)$$

Linear Equations of Motion

$$y(1) = .01 x(3) x(1) + [x(4) -10 x(5)] [x(2) - u(1)] + 10 x(5) x(2) \quad (6.21)$$

$$y(2) = x(1)$$

Equations (6.19), (6.20) and (6.21) were tested varying the following parameters:

1. Initial State Estimate - The "true values" of the parameters ($x(3)$ through $x(7)$) were set in the simulation at the values given by Equation (6.17). The initial estimate was set either zero or Equation (6.17).

2. Initial State Estimate Error Covariance P_0 - This is a diagonal matrix of equal elements which were set to 0, 2, and 5.

3. Order of Magnitude of the Elements of R Relative to the Elements of Q - Both these matrices are diagonal and of equal elements. The ratio of the elements of R to the elements of Q was set at 1, 2, 5 and 10.

4. Noise Level in Environmental Noise - \underline{w} and in Measurement Noise - \underline{v} . Set at 0 and 1

Then the collected data was used in the identification procedure introducing small variations in the parameters mentioned in 1, 2 and 3 above around a reasonable set

obtained in the simulation to obtain the best possible convergence and also to test the effect of initial conditions - or in other words, the necessary accuracy in the theoretical prediction of the values of the parameters, to obtain convergence. Some of the results from the identification are presented in Appendix 5 while the discussion of the results is presented in Chapter 7.

Summarizing in this chapter some of the available techniques for parameter identification were discussed. Attention was focused in those that permit application to non-linear dynamic systems. From those, one was chosen that having perhaps less accuracy in the results, is easy to implement. The equations of motion for roll went then through the necessary modifications to be in agreement with the format of such formulation. The structure of the model and the formulation were tested by simulating the excitation and the measurements, and varying selected parameters to obtain a better insight of what to expect from the identification.

CHAPTER 7

DISCUSSION OF RESULTS

1. Simulation

The first equation to be tested was Equation 6.19. The noise level was set to zero, the ratio of elements of \underline{R} to elements of \underline{Q} , from now on expressed formally as r/q by similarity with the scalar case, equal to $\underline{1}$, the initial state estimate error covariance matrix \underline{P}_0 at [2] and the initial state estimate x_0 at zero (the "true value" was set in all simulations at the values expressed in equation (6.17)). The values of the estimate blew up and after 200 measurements the computer indicated overflow. This type of behaviour with no noise decided the use of Equations (6.20) or (6.21). These two models were both tested at no noise. Equations (6.20) were first tested with $\underline{x}(0) = \text{Eq.}(6.17)$, $r/q = 1$ and $\underline{P}_0 = [5]$. The error on the estimate ($\underline{XHAT} - \underline{XX}$) where \underline{XHAT} is the estimate and \underline{XX} is the "true value" of the state, remained at zero during the whole experiment. However $P(3,3)$ (variance of $x(3)$ - linear damping coefficient) kept increasing with small oscillations. The other elements of \underline{P} decreased quickly to less than .1 . The same behaviour was observed for Equations (6.21), showing that the influence of the

non-linear terms is small, at least for the no noise condition.

The last experiment in the no noise series was conducted with Equations (6.21) $\underline{P}_0 = [2]$, $r/q = 1$ and $(\underline{XHAT})_0 = [0]$.

The error $(\underline{XHAT} - \underline{XX})$ for all variables converged rapidly to zero - in less than 100 measurements with the exception of $(\underline{XHAT}(3) - \underline{XX}(3))$ whose value remained constant throughout. Again the term $P(3,3)$ steadily increased; while all the other variances came to values, after 500 measurements, less than .060 (initial value was 2.0) the final value of $P(3,3)$ was 2.202. The foregoing suggests that when no noise is present the system converges independently of initial conditions, with the exception of Equations (6.19) and with the exception of the linear damping coefficient $x(3)$ whose identification seems very difficult to obtain.

The rest of the experiments were performed with noise level equal to 1 which is believed to be quite a high level due to the scattering observed in the measurement vector \underline{z} .

The first simulations were taken with $r/q = 1$ $\underline{XHAT}_0 = [0]$ and $\underline{P}_0 = [3]$ and $= [5]$. Both simulations blew up at about the 140th measurement. At that time $\underline{P}(3,3)$,

$P(4,4)$ and $P(5,5)$ were quite large, but different from one case to the other. The main point to be observed here is the conjugate effect of noise and not accurate initial estimate.

The next experiment put together Equations (6.20) and (6.21). For both systems $r/q = 1$, $\underline{P}_0 = [0]$ and $(\underline{XHAT})_0 = (\underline{XX})_0$. Since initially the error covariance matrix was set to zero, it increased and then oscillated about values equivalent to the ones found in the no noise condition (less than .050), with the exception of $P(3,3)$ which steadily increased to a final value at the 500th measurement of .249 in both cases, with tendency to increase. The errors on the variables oscillates around zero with the exception of $(\underline{XHAT}(3) - \underline{XX}(3))$ which remained constant at about .035. Again it seems that the identification of $x(3)$ will be difficult to obtain.

Next an experiment was performed with Eq. (6.20) with $r/q = 1$, $(\underline{XHAT})_0 = (\underline{XX})_0$ but $\underline{P}_0 = [5]$ which diverged again confirming the influence of the initial conditions when combined with noise.

The next set of experiments were performed with the noise level at 1, as before, but $r/q = 2, 5$ and 10.

Comparing the results from Equations (6.21) for

$r/q = 1$ and $r/q = 2$, $(\underline{XHAT})_0 = [0]$ and $\underline{P}_0 = [2]$, in the later case the estimate does not blow up although the error on the estimate and the variance both increase for $x(3)$ and $x(4)$, while for $x(5)$ the error on the estimate and the variance decreases, which demonstrates some improvement. The cases $r/q = 5$ $r/q = 10$ represent no advantage over $r/q = 2$, presenting the same trends.

Similar simulation was performed with Equations (6.20). Again an improvement is noted when r/q goes up to two but no further improvement is obtained when $r/q = 5$ or 10 . On the contrary, the decreasing in the error seems to take place more slowly in the later case. In the case $r/q = 2$ the advantage of considering the non-linear terms when identifying the linear parameters is noted because $P(3,3)$, which increased in the case of Equations (6.21), is seen to oscillate between approximately 2.02 and 2.16, and $P(4,4)$, after increasing for the first couple of measurement, markedly decreases. The foregoing results were then compared with the case of $\underline{P}_0 = [5]$, with clear advantage for the former.

The foregoing seems to indicate that the values $r/q = 2$ and $\underline{P}_0 = [2]$ are the best suited among all other combinations tested. At this stage small variations could be tried to further improve the convergence.

But since the characteristics of the data are not expected to follow very closely the characteristics of

the simulation, it seems a better idea to try to identify the linear parameters with the data obtained from the experiments and then vary the values of \underline{R} , \underline{Q} and \underline{P}_0 in order to obtain a better convergence. But before that, one last simulation was tried for the same values of \underline{Q} , \underline{R} and \underline{P}_0 , but for values of $(\underline{XHAT})_0$ closer to the true values : $(\underline{XHAT})_0 = [0, 0, -.2, -.5, .15]$ (about 1/2 of the true values). The number of measurements was also increased to 1000. It was noted that :

i) $P(5,5)$ decreased rapidly to a value of .4 (after 10 measurements) and then decreased oscillating until it reached a value of .057. The error on the estimate, after 1000 measurements, was oscillating between $\pm .1$.

ii) $P(4,4)$ decreased less rapidly to a value of .566 (after 30 measurements) and then jumped to a value of .06 where it kept oscillating for about 350 measurements decreasing finally to a value of .025 where it stayed oscillating until the end. The error on the estimate oscillated between ± 1 for 400 measurements and then decreased to zero oscillating about that point.

iii) $P(3,3)$ increased to a value of 2.22 after 400 measurements, oscillated for a while and finally decreased to reach at the end a value of 1.29, with tendency for further decrease. The error on the estimate

increased to a value of 3.3 after 420 measurements, oscillated about that point for a while and finally decreased, oscillating to a value of 1 at the end.

Note that the turnover point seems to be the 400th measurement and that a set of initial conditions on the estimate closer to the true value really improves convergence. This fact confirms the believe that, contrary to the linear case, the initial conditions on both the estimate and its covariance play an important in the success or failure of the identification.

2. Identification Using Experimental Data

Following the conclusions of last section a run was made with $\underline{Q} = [.1]$, $\underline{R} = [.2]$ and $\underline{P}_0 = [2.]$. The set of initial conditions was as prescribed by Equations (6.17). Simultaneously, runs with the values of \underline{Q} , \underline{R} and \underline{P}_0 as indicated below were also made. The set of initial conditions, however, was not changed at this stage. It seemed preferable, since the above set is believed to be, among the available predictions, the closest to the true value, to find first the best set of \underline{Q} , \underline{R} , and \underline{P}_0 changing $(\underline{XHAT})_0$ afterwards.

Values of \underline{Q} , \underline{R} and \underline{P}_0 :

$$\underline{Q} = [.1]$$

$$\underline{R} = [.1] , [.2] , [.3]$$

$$\underline{P}_0 = [1.] , [2.] , [5.]$$

in the following combinations of \underline{R} and \underline{P}_0 :

([.05] , [2.])

([.1] , [2.]) , ([.1] , [5.])

([.2] , [1.]) , ([.2] , [2.]) , ([.2] , [5.])

([.3] , [1.])

For the group ([.2] , [2.]) the results obtained with the first measurements were more or less as expected. $P(5,5)$ decreased rapidly, $P(4,4)$ decreased less rapidly and $P(3,3)$ increased steadily. Then $P(4,4)$ came close to zero and became negative for a while, then crossed zero again and went up to 2.48, decreased to .270 and increased again to the final value of .421. Also at the end $P(5,5) = .059$ and $P(3,3) = 2.473$. On the other runs a similar behaviour was noted. For $\underline{R} = [.2]$ and $\underline{P}_0 = [1.]$, $P(4,4)$ crossed zero and kept decreasing up to a final value of -1.589. $P(5,5)$ decreased to reach the value of .064 (at the same point at which $P(4,4)$ became negative) and then increased to reach a value of .123 at the end. $P(3,3)$ increased steadily to reach 1.462 at the end. For the case $\underline{R} = [.2]$, $\underline{P}_0 = [5.]$ it was even worse since at the end $P(3,3) = -43.225$ and $XHAT(3) = 252.70$. For $\underline{R} = [.1]$, $\underline{P}_0 = [5.]$, $P(3,3)$ increased steadily, $P(4,4)$ decreased to a value of .173 and then increased again to reach at the end .425 with $XHAT = -14.513$ (?). $P(5,5)$ decreased to a value of .028 and kept oscilla -

ting about that value, but with $XHAT(5)$ always increasing. For $\underline{R} = [.1]$, $\underline{P}_0 = [2.]$ the behaviour was similar but not so extreme due to the smaller value of \underline{P}_0 .

For $\underline{R} = [.05]$, $\underline{P}_0 = [2.]$, $P(3,3)$ always increased while $P(4,4)$ and $P(5,5)$ decreased first to low values (.60 and .20) and increased afterwards, never becoming negative. For the last run of this group ($\underline{R} = [.3]$, $\underline{P}_0 = [1.]$), the type of behaviour was similar to ($\underline{R} = [.2]$, $\underline{P}_0 = [1.]$) but much more extreme — the last recorded value of $P(4,4)$ was -226.1 at the 350th measurement because some cycles later the computer stopped by overflow.

The fact that $P(4,4)$ became negative posed the question of the correctness of the computerized scheme. However the program after thorough checking have been working successfully with linear systems. Besides no such strange behaviour was found during the simulation.

Therefore the program was checked again in detail but nothing could be found able to explain the foregoing.

It was suggested then that as $P(4,4)$ approaches zero, the computer could introduce some errors thus making it negative. The process would be selfsustaining afterwards. Assuming that to be the cause, the solution proposed was, whenever $P(I,I)$ became negative, to substitute it by its positive counterpart. Accordingly

the following statements were introduced in the Kalman filter (KLMN subroutine) :

```
DO 21 I 1, NX
  IF( P(I,I).LT.0.0) P(I,I)= -P(I,I)
21 CONTINUE
```

The effects of this procedure were then verified on the following combinations of \underline{R} and \underline{P}_0 :

```
( [.2] , [ 1.] )
( [.2] , [ 5.] )
( [.3] , [11.] )
```

For the second case, this had as a consequence to hold XHAT(4) within bounds longer than before, only to diverge more violently towards the end (XHAT(4) = 735.). The other two parameters had also large values. For the first case, the influence of the above procedure is more evident, since originally XHAT(4) decreased, crossed zero at approximately the 100th measurement (point where P(4,4) became negative) and went on decreasing. With the presence of the above statements, it behaved in the same way for the first 100 measurements, then stayed at a value of .8 until the 300th measurement, starting a slow decrease through zero to reach the value of -2.605 at the end . Accordingly the variance P(4,4), after touching zero increased to a final value of .242 . The same type of behaviour is also noted in

both XHAT(3) — diverging with negative values before, with P(3,3) increasing and being constant now with P(3,3) increasing also, and XHAT(5). — going away from zero with positive values before and going away from zero but with negative values and at a much slower rate, with the introduction of the above statements. P(4,4) seemed to be bouncing between zero and some finite positive values, but since the values of P were only recorded every 50 measurements, no valid conclusion can be taken about that fact. P(3,3) always increased and P(5,5) remained at approximately .057 .

It was decided then to modify the structure of the state equations to check if another form of the equations would converge to a solution. In Equation (6.16), for example the two groups forming the coefficient of $x(2) - u(1)$ could be considered as one parameter, being the new forms of $x(4)$ and $x(5)$:

$$\begin{aligned} 10 x(4) &= \frac{K - W.BG}{-K_p + I_{xx}} \cdot \frac{1}{\omega^2} \\ 10 x(5) &= \frac{W.BG}{-K_p + I_{xx}} \cdot \frac{1}{\omega^2} \end{aligned} \quad (7.1)$$

And the new form of the state equations would then be :

$$\begin{aligned} y(1) &= .01 x(3) x(1) + x(4) [x(2) - u(1)] \\ &+ 10 x(5) x(2) + .0126 x(1) \text{ABS}(x(1)) \\ &+ [.0312 + .0167 x(5)] [x(2) - u(1)]^3 \\ &- .0167 x(5) x(2)^3 \end{aligned} \quad (7.2)$$

$$y(2) = x(1)$$

For comparison only the following values of \underline{R} and \underline{P}_0 were considered :

i) $\underline{R} = [.2]$ and $\underline{P}_0 = [5.]$

ii) $\underline{R} = [.3]$ and $\underline{P}_0 = [1.]$

Also and to check the behaviour of the system at high values of \underline{Q} and \underline{R} , the following run was made :

iii) $\underline{Q} = [1.]$, $\underline{R} = [2.]$, $\underline{P}_0 = [1.]$

The main characteristics of these runs was that the parameters did not oscillate, or oscillated with a very long period. Only in the case $\underline{R} = [.2]$, $\underline{P}_0 = [5]$, XHAT(3) went over 1. at the 310th measurement, approximately. XHAT(4) and XHAT(5) which started at -.373 and +.274 respectively, crossed each other and zero at the 520th measurement reaching at the end the values +.359 and -.422 and showing in between a behaviour completely linear. P(3,3) increased to a value of 5.034 at the 100th measurement and then decreased as steadily as it had increased to a final value of 3.412. Similarly, P(4,4) increased to a value of 5.003 in the first 15 measurements and then decreased to a final value of 4.838. P(5,5) from the start decreased oscillating until it reached a final value of .384 . This looked like being the object of the search apart from the fact that the values the parameters reached at the

end were not to be expected on physical grounds, specially $XHAT(5)$ (this would mean that the center of bouyancy had to be above the center of gravity which is not true from the calculated curves). A possibility to consider is that the variables go through a cycle of very long period and therefore the convergence will take place later in the process. For the case $R \quad [.3] \quad , \quad P_0 \quad [1.] \quad ,$ only the value of $P(5,5)$ decreased and the parameters changed their value very slowly and therefore nothing could be said about such a process. It could be an oscillatory motion of a period even longer than before or it could be a steady, but very slowly divergent process. In the last case considered, the parameters did not change but the diagonal elements of the covariance matrix increased and quite rapidly to reach values of $[1.91 \quad , \quad 2.239 \quad , \quad 48.289 \quad , \quad 48.195 \quad , \quad 32.620]$. Note that that the variance of $XHAT(1)$ and $XHAT(2)$ was, for all the cases considered, simulation included, of the order of .025 and .012 after the first 20 to 30 measurements. Therefore this last case is of no consequence.

In summary in this chapter the results from the identification were presented, both for the simulation carried out and for use of the data. It was pointed out of the difficulty of the system to converge. A set of values of the different covariance matrices was selected

from the simulation studies as being those that offered more possibilities when using real data. However the results didn't meet the expectations, being necessary to proceed in looking a set that would make the system converge . In stead it was found that some of the diagonal terms of \underline{P} would become negative. First it was thought to be resultant from a mistake in the computerised scheme and later to errors introduced by the computer itself and corrected accordingly. Even then no solution could be found after 1000 measurements . Then another form of the state equations was obtained by combining two of the parameters. This later system of equations presented some interesting characteristics, but nothing could be said in definite.

CHAPTER 8

CONCLUSIONS AND RECOMENDATIONS

From Chapter 7 it was quite clear that convergence of the estimate of the system to its true value was, in general, not possible to obtain. Reasons that could account for this failure in convergence were then pointed out as follows :

- 1 - Possible mistakes in the computerized scheme.
- 2 - Not accurate prediction of the initial conditions of the system.
- 3 - Data that measured the dynamical behaviour was not accurate enough.
- 4 - The input to the model didn't really have the same characteristics as the input to the system in the experiments.

To those reasons above another may be pointed out that could be partly responsible for the non-convergence of the system, namely :

- 5 - Incompatibility between the system structure and the particular technique that was chosen .

With respect to the first reason, the fact , mentioned already that this same procedure was used before in the identification of linear static systems successfully, although not a sufficient reason to

completely exclude this possibility, at least decreases the probability of such an event. Therefore it is recommended that if future use is intended to such a formulation, the program be rechecked for mistakes.

The not accurate prediction of the initial conditions can be a reason of some standing if the methods used for prediction fall under the category of empirical or semiempirical formulations from which the final result have not the reliability required. For the calculation of the motion parameters for roll, for example, the three categories in which Chapter 3 was divided also provide a good basis for classification of accuracy, starting with the hydrostatic coefficients at the top and the non-linear coefficients at the bottom of the scale. It is believed that for the case of excitation due to regular waves a formulation as presented by Ref. 18 and 19 for linear added mass, added moment of inertia and damping coefficient is completely adequate in the sense that the system will converge if the values of those parameters constitute the only approximation involved. This conclusion is reached with information on the simulations carried out in Chapter 7.

When trying to digitize the data from the towing tank experiments, it was believed that this item

played a major role in the success or failure of the identification. This is not so anymore, when one compares the data from the tank with the measurements generated in the simulation, where the " level " of noise is much higher than in the former case. This is not the same as saying that it is not necessary to care about the experiments. If nothing else a careful planning will save a lot of time and labor, waisted otherwise trying to correct past mistakes.

The accurate determination of the input to the system as a function of time seems to play an important role in the success of the identification. Intuitively if the behaviour of two systems is to be compared, it is necessary that the input be the same. Unfortunately this was not the case. Both methods presented in Chapters 3 and 4 (using the Haskind Relations or assuming the major part of the excitation to be due to changes in bouyancy due to the passage of the wave), will not give very accurate results, the first because it does not present the instantaneous values of the input, but an expression that assumes before hand an harmonic tims dependency, the second because, although it expresses the input as the instantaneous function of a quantity being measured in the experiment, exprimes a very rough approximation of such dependence. However

the second was used in the identification procedure and it is still believed that, for this particular case the second yields better results.

The incompatibility between the system structure and the method of identification exists because some of the assumptions set forth in the derivation of this particular method of identification were not met by the system. One of such assumptions was that the noise in the system, both environmental and measurement noise were uncorrelated with each other and with the state of the system (Equations 6.5 and 6.6). This last point was not met by the vehicle structure since, for example, finite water depth effects, interference between the vehicle and the walls, etc., would in one way or another be correlated with motions of the vehicle. This fact as a reason for failure to achieve convergence gained importance when comparing the results of the simulation with the results of the identification, where the former, being bad, were a bit more encouraging than the last.

With the above in mind, it seems that in order to improve the application of the identification techniques to problems of the type discussed herein, it will be necessary :

- 1 - Generate a vehicle environment that can be

better approximated as an uncorrelated random process. This suggests the possibility of using random seas to generate the data for the identification.

2 - Use another method of system identification, such as the model reference technique briefly described in Chapter 6, which do not make such restrictive assumptions.

BIBLIOGRAPHY

1. F. Galiana, "A Review of Basic Principles and of Available Techniques in System Identification," M.I.T. Power Systems Engineering Group Report No. 20, Nov. (1969).
2. M. N. Hayes, "Request for Thesis Subject Approval," Jan. 22 (1971).
3. J. H. Vugts, "The Hydrodynamic Forces and Ship Motions in Waves," University of Delft (1970).
4. M. A. Abkowitz, "Stability and Motion Control of Ocean Vehicles," M.I.T. Press (1969).
5. W. H. Tufts, "Emergency Ascent Trajectories for Deep Submersibles," M.I.T. Department of Naval Architecture and Marine Engineering,, NavE Thesis, June (1969).
6. J. E. Kerwin, "Polynomial Surface Representation of Arbitrary Ship Forms," Journal of Ship Research, Vol.4, No. 1, June (1960).
7. G. G. Parissis, "The New Program for Kerwin's Polynomial Representation Ship Forms," M.I.T. Contract DSR 5054, June (1965).
8. W. R. Porter, "Pressure Distribution, Added Mass and Damping Coefficients for Cyclinders Oscillating in a Free Surface," University of California - Inst. Eng. Research, July (1960).

9. J. Kotik and V. Mangulis, "On the Kramer-Kronig Relations for Ship Motions," International Ship Building Progress, Vol. 9, No. 97, September (1962).
10. T. Ogilvie, "Recent Progress Toward the Understanding and Predictions of Ship Motions," Proceedings of the Fifth Symposium on Naval Hydrodynamics, Bergen, Norway, September (1964).
11. J. N. Newman, "The Exciting Forces on Fixed Bodies in Waves," Journal of Ship Research, Vol. 6, No. 3, December (1962).
12. J. N. Newman, "The Damping of an Oscillating Ellipsoid Near a Free Surface," Journal of Ship Research, Vol. 5, No. 3, December (1961).
13. B. V. Korvin-Kroukovsky, "Investigation of Ship Motions in Regular Waves," SNAME Transactions, Vol. 63 (1955).
14. D. V. Korvin-Kroukovsky and W. R. Jacobs, "Pitch and Heaving Motions of a Ship in Regular Waves," SNAME Transactions, Vol. 65 (1957).
15. B. V. Korvin-Kroukovsky, "Theory of Seakeeping," SNAME (1961).
16. J. H. Vugts, "Hydrodynamic Coefficients for Swaying, Heaving and Rolling Cylinders in a Free Surface," Technische Hogeschool Delft Report 194, January (1968).

17. R. T. Bermejo, "Added Mass and Damping Coefficients for Ships Heaving in Smooth Water," M.I.T. Department of Naval Architecture and Marine Engineering Report 65-5, June (1965).
18. W. Frank, "Oscillation of Cylinders in or Below the Free Surface of Deep Fluids," NSRDC Report 2375, October (1967).
19. J. Bedel and C. Lee, "Numerical Calculation of the Added Mass and Damping Coefficients of Cylinders Oscillating in or Below a Free Surface," NSRDC Report 3551, March (1971).
20. S. N. Blagoveshchensky, "Theory of Ship Motions," Dover Publications (1962).
21. D. H. Hines and L. C. Gies, "Experimental Determination of Ship Motion Parameters by a Step Response Technique for a Mariner Class Hull Form in Pitch and Heave," M.I.T. Department of Naval Architecture and Marine Engineering Report 65-4, June (1965).
22. S. Tamiya and S. Motora, "Advances in Research on Stability and Rolling of Ships," The Society of Naval Architects of Japan 60th Anniversary Series, Vol. 6, (1960).

S. Tamiya, "Calculation of Non-linear Non-symmetric Rolling of Ships," 12th International Towing Tank Conference.

23. S. Goldstein, "Modern Developments in Fluid Dynamics," Vol. II, P. 424-426.
24. J. N. Newman, "Lecture Notes on Marine Hydrodynamics," unpublished.
25. H. Luft, "Wave Probes for Model Tanks," G.M.B.H. Report No. F46 167, translated from German by S. D. Sharma and W. H. Roth for the Department of Naval Architecture and Marine Engineering, University of Michigan.
26. 680 Scientific Computing System Reference Handbook, Electronics Associates.
27. J. Gerristma and W. Beukelman, "The Distribution of Hydrodynamic Forces on a Heaving and Pitched Ship Model in Still Water," International Ship-building Progress, Vol. 11, No. 123, November (1964).
28. J. E. Kerwin and H. Narita, "Determination of Ship Motion Parameters by a Step Response Technique," Journal of Ship Research, Vol. 9, No. 3, December (1965).
29. A. E. Bryson and Y. C. Ho, "Applied Optimal Control," Blaisdell Publishing Company, (1969).

APPENDIX 1

HYDROSTATIC COEFFICIENTS

- 1 - DISCRIPTION OF THE COMPUTER PROGRAMS TO OBTAIN A POLYNOMIAL SURFACE REPRESENTATION
- 2 - VALUES OF THE HYDROSTATIC COEFFICIENTS FOR A MODEL OF THE " MARINER " HULL FORM

POLYNOMIAL SURFACE REPRESENTATION USING LEGENDRE POLYNOMIALS
 IMPLEMENTED BY G. PARISSIS BASED ON KERWIN'S METHOD
 DISCRPTION AND FORMAT OF INPUT PARAMETERS

```

CARD 1      FORMAT(3I20)
          IT - NUMBER OF DATA POINTS PER CURVE ( MAX = 60 )
          JT - TERMS IN THE POLYNOMIAL IN Z ( MAX = 20 )
          MT - NUMBER OF CURVES TO BE FITTED ( MAX = 25 )

CARD 2      FORMAT(2F20.5)
          X1 - LOCATION OF FIRST DATA POINT
          -0.25L0E0X1L0E0.0 IN THE NONDIMENSIONAL INTERVAL ( 0,1 )
          X1 = 1.0 IN THE NONDIMENSIONAL INTERVAL ( -1,1 )
          X2 - LOCATION OF LAST DATA POINT
          USUALLY X2 = 1.0

CARD 3      FORMAT(3I20)
          K(N) - LOCATION OF SPECIAL TERMS IN Z - N = 1,2,3
          THE SPECIAL TERMS SHOULD ALWAYS BE THE LAST TERMS IN THE Z
          POLYNOMIAL. THUS, IF JT = 10 AND THERE ARE THREE SPECIAL
          TERMS: K(1) = 8, K(2) = 9, K(3) = 10, IF JT = 10
          AND THERE ARE TWO SPECIAL TERMS: K(1) = 9, K(2) = 10 AND
          THERE IS NO K(3). IF THERE ARE NO SPECIAL TERMS THIS CARD
          REMAINS BLANK

CARD 4      FORMAT(3I20)
          L(N) - EXPONENTS OF SPECIAL TERMS IN Z - N = 1,2,3
          MAX. NUMBER = JT. OTHERWISE NO RESTRICTION. WHEN THERE ARE
          LESS THAN THREE SPECIAL TERMS, WHAT WAS SAID IN CARD 3
          APPLIES

CARD 5      AND FOLLOWING CARDS      FORMAT(11F6.4)      - DATA POINTS
          DATA(I,M)      I=1, IT      M=1, MT
          INDICATE THE VALUES OF THE SURFACE TO BE FITTED STARTING AT
          LOCATION X1 AND ENDING AT LOCATION X2 ( FIRST IN THE Z DI-
          RECTION AND THEN IN THE FI DIRECTION )
          JT - NUMBER OF TERMS IN THE POLYNOMIAL IN X ( MAX. = 20 )
          CARD FOLLOWING THE LAST DATA CARD ( CARD K )      FORMAT(I20)
          CARD R+1      FORMAT (2F20.5)
          X1 - LOCATION OF FIRST DATA POINT      X1 = 1.0
          X2 - LOCATION OF LAST DATA POINT - X2 = -1.0
  
```



```

C      CARD  R+2      FORMAT(3I20)
C      K(N)  - LOCATION OF SPECIAL TERMS IN FI - N = 1,2,3
C      SAME COMMENTS AS FOR CARD 3
C      CARD  R+3      FORMAT(3I20)
C      L(N)  - EXPONENTS OF SPECIAL TERMS IN FI - N = 1,2,3
C      SAME COMMENTS AS FOR CARD 4
C      DIMENSION K(3),L(3),DATA(28,28),A(28,20),H(28,28),D(20,20),C(20,28
1) ,UNIT(20,20),R(28),SPACE(20),B(20,20),G(20,25),Q(20,25),E(28,20),
2POLYN(25,25),T(20,20)
200 FORMAT(73H *NO OF DATA PER CURVE* *TERMS IN POLYNOMIAL OF Z* *END
1OF CURVES TO FIT*)
201 FORMAT(3I20)
202 FORMAT(63H *LOCATION OF FIRST DATA POINT* * LOCATION OF LAST DATA
1POINT*)
203 FORMAT(2F20.5)
204 FURMAT(29H *LOCATION OF SPECIAL TERMS*)
205 FURMAT(30H *EXPONENTS OF SPECIAL TERMS*)
206 FORMAT(2H ///8H *DATA*)
207 FORMAT(11F6.4)
208 FORMAT(2H ///68H *EVALUATION OF LEGENDRE POLYNOMIALS IN Z AT THE
1INPUT DATA POINTS*)
209 FORMAT(2H ///59H *EVALUATION OF COEFFICIENTS OF LEGENDRE POLYNOM
1IALS IN Z*)
210 FORMAT(4X,10E12.6)
211 FURMAT(2H ///42H *RMS ERROR FOR EACH OF THE ABOVE CURVES*)
212 FORMAT(10X,E12.6)
213 FURMAT(2H ///72H *COEFFICIENTS OF POWERS OF Z STARTING WITH ZERO
1ETH POWER AND GOING UP*)
214 FURMAT(2H ///34H *TERMS IN THE POLYNOMIAL OF FI*)
215 FURMAT(2H ///45H *COEFFICIENTS OF LEGENDRE POLYNOMIALS IN Z*)
216 FURMAT(2H ///108H *MATRIX OF COEFFICIENTS OF Z AND FI TERMS RE
1AD INCREASING POWERS OF Z HORIZONTALLY AND OF FI VERTICALLY*)
217 FORMAT(1I20)
218 FORMAT(11F10.4)
219 FURMAT(2H ///24H *TEST PRINT OF Q(J,M)*)
222 FURMAT(2H ///73H *COEFFICIENTS OF LEGENDRE POLYNOMIALS A(M,N). N

```



```

1 INCREASES HORIZONTALLY*
  WRITE(6,200)
300 READ(5,201)IT,JT,MT
  IF(IT)550,550,301
301 WRITE(6,201)IT,JT,MT
  WRITE(6,202)
  READ(5,203)X1,X2
  WRITE(6,203)X1,X2
  WRITE(6,204)
  READ(5,201)(K(N),N=1,3)
  WRITE(6,201)(K(N),N=1,3)
  WRITE(6,205)
  READ(5,201)(L(N),N=1,3)
  WRITE(6,201)(L(N),N=1,3)
  IF(ABS(X2-X1)-1.5)500,500,501
500 INTVAL=1
  GO TO 504
501 INTVAL=2
504 WRITE(6,206)
  READ(5,207)((DATA(I,M),I=1,IT),M=1,MT)
  WRITE(6,218)((DATA(I,M),I=1,IT),M=1,MT)
516 CALL HUPOL2(IT,JT,MT,K,L,X1,X2,INTVAL,DATA,A,H,D,C,UNIT,E,R)
  WRITE(6,208)
  WRITE(6,218)((H(I,M),I=1,IT),M=1,MT)
  WRITE(6,211)
  WRITE(6,212)(R(M),M=1,MT)
  WRITE(6,215)
  DO 610 M=1,MT
610 WRITE(6,210)(C(J,M),J=1,JT)
  IF(K(1))510,510,511
510 JJT=JT
  GO TO 512
511 JJT=K(1)-1
512 CALL LEGTR(JJT,INTVAL,B)
  DO 507 KK=1,JJT
  DO 507 MM=1,MT

```



```
TERM=0.0
DO 506 LL=1, JJT
506 TERM=TERM+C(LL,MM)*B(LL, KK)
507 G(KK,MM)=TERM
K1=K(1)
K2=K(2)
K3=K(3)
INT1=INTVAL
IF(K(1))522,522,514
514 DO 508 MM=1,MT
G(K1,MM)=C(K1,MM)
G(K2,MM)=C(K2,MM)
508 G(K3,MM)=C(K3,MM)
522 WRITE(6,213)
DO 611 MM=1,MT
611 WRITE(6,210)(G(KK,MM),KK=1,JT)
MT1=MT
JT1=JT
IT=MT
MT=JT
WRITE(6,214)
READ(5,217)JT
IF(GT)550,550,525
525 WRITE(6,217)JT
WRITE(6,202)
READ(5,203)X1,X2
WRITE(6,203)X1,X2
WRITE(6,204)
READ(5,201)(K(N),N=1,3)
WRITE(6,201)(K(N),N=1,3)
WRITE(6,205)
READ(5,201)(L(N),N=1,3)
WRITE(6,201)(L(N),N=1,3)
IF(ABS(X2-X1)-1.5)540,540,541
540 INTVAL=1
GO TO 542
```



```

541 INTVAL=2
542 DO 505 M=1,MT
543   DO 505 I=1,IT
544     CATA(I,M)=C(M,I)
545     CALL HUPOL2(IT,JT,MT,K,L,X1,X2,INTVAL,DATA,A,H,D,C,UNIT,E,R)
546     WRITE(6,209)
547   DO 612 I=1,IT
548     WRITE(6,210)(H(I,M),M=1,MT)
549     WRITE(6,211)
550     WRITE(6,212)(R(M),M=1,MT)
551     WRITE(6,222)
552   DO 613 M=1,MT
553     WRITE(6,210)(C(J,M),J=1,JT)
554     IF(K(1))531,531,532
555   JJT=JT
556   GO TO 533
557   JJT=K(1)-1
558   CALL LEGTR(JJT,INTVAL,T)
559   DO 535 KK=1,JJT
560     DO 535 M=1,MT
561       TERM=0.0
562     DO 534 LL=1,JJT
563       TERM=TERM+C(LL,M)*T(LL,KK)
564     Q(KK,M)=TERM
565     IF(K(1))536,536,537
566     KKL=K(1)
567     KK2=K(2)
568     KK3=K(3)
569     DO 538 M=1,MT
570       Q(KKL,M)=C(KKL,M)
571       Q(KK2,M)=C(KK2,M)
572       Q(KK3,M)=C(KK3,M)
573     WRITE(6,219)
574     WRITE(6,210)((Q(J,M),J=1,JT),M=1,MT)
575     IF(K1)570,570,571
576     MMT=MT

```



```
GO TO 572
571 MMT=K1-1
572 DO 574 MK=1,MMT
DO 574 JJ=1,JT
TERM=0.0
DO 573 ML=1,MT
573 TERM=TERM+Q(JJ,ML)*B(ML,MK)
574 POLYN(JJ,MK)=TERM
IF(K1)576,576,577
577 DO 578 J=1,JT
POLYN(J,K1)=Q(J,K1)
POLYN(J,K2)=Q(J,K2)
578 POLYN(J,K3)=Q(J,K3)
576 WRITE(6,216)
DO 579 JX=1,JT
WRITE(6,210)(POLYN(JX,MZ),MZ=1,MT)
579 CONTINUE
GO TO 300
550 CALL EXIT
END
```



```

C      20 SEPTEMBER 1966
SUBROUTINE HUPOL2(IT,JT,MT,K,L,X1,X2,INTVAL,DATA,A,H,D,C,UNIT,E,R)
DIMENSION K(3),L(5),DATA(28,28),A(28,20),H(28,28),D(20,20),
1 C(20,28),UNIT(20,20),E(28,20),R(28),SPACE(20)
C THIS PROGRAM COMPUTES MODIFIED LEGENDRE POLYNOMIAL APPROXIMATIONS
C H(X) TO UP TO 28 SETS OF INPUT DATA DATA(X). EACH CURVE MAY HAVE
C UP TO 28 POINTS AND THE MAXIMUM NUMBER OF TERMS IS 20. ANY 3
C OF THESE TERMS MAY HAVE ARBITRARY EXPONENTS SPECIFIED.
C
C PROVIDED BY CALLING PROGRAM      ARGUMENTS      COMPUTED BY HUPOL2
C IT=NUMBER OF DATA POINTS/CURVE      A=COORDINATE FUNCTIONS
C JT=TERMS IN POLYNOMIAL              H=POLYNOMIAL EVALUATED
C MT=NUMBER OF CURVES TO FIT          D=NORMAL EQUATIONS AND INVERSE
C K=LOCATION OF SPECIAL TERMS          C=COEFFS OF LEGENDRE POLYNOMIALS
C L=EXPONENTS OF SPECIAL TERMS        UNIT=UNIT MATRIX TO GET INVERSE
C X1=X VALUE OF FIRST DATA POINT     E=RIGHT HAND SIDE OF NORMAL EQNS
C X2=X VALUE OF LAST DATA POINT       R=RMS ERROR FOR EACH CURVE
C INTVAL=1 IF X RANGE NEAR (0,1)
C      2 IF X RANGE NEAR (-1,1)
C DATA=INPUT DATA TO BE FITTED
C
C      GENERATE COORDINATE FUNCTIONS
C      QIT=FLOAT(IT)
C      DELT=(X2-X1)/FLOAT(IT-1)
C      ASSUME FIRST ALL ARE LEGENDRE
C
C      DO 1 I=1,IT
C          Z=X1+DELT*FLOAT(I-1)
C          A(I,1)=1.0
C          GO TO (14,15),INTVAL
C          A(I,2)=2.0-Z-1.0
C
C          GO TO 16
C          A(I,2)=Z
C          DO 2 J=3,JT
C              M=J-1
C              A(I,J)=(FLOAT(2*M-1)*A(1,2)*A(I,J-1)-FLOAT(M-1)*
1              A(I,J-2))/FLOAT(M)

```



```

2  CONTINUE
C
C      REPLACE THE FOLLOWING LEGENDRE TERMS
      WITH SPECIAL TERMS

4      DO 3  M=1,3
          IF(K(M))      3,3,4
          LL=L(M)
          KK=K(M)
          A(I, KK)=Z**LL

3  CONTINUE
1  CONTINUE
C

      FORM NORMAL EQUATIONS

      DO 8  M=1, JT
      DO 9  N=1, M
          D(M, N)=0.0
          UNIT(M, N)=0.0
          UNIT(N, M)=0.0

      DO 10 I=1, IT
          D(M, N)=D(M, N)+A(I, M)*A(I, N)

10  CONTINUE
      IF(M=N)  11, 17, 11
          D(N, M)=D(M, N)

11  GO TO 9

17  UNIT(M, N)=1.0
5  CONTINUE
8  CONTINUE
      DO 12 M=1, MT
      DO 12 N=1, JT
          E(M, N)=C.0

      DO 12 I=1, IT
          E(M, N)=E(M, N)+A(I, N)*DATA(I, M)

12  CONTINUE
C  GET INVERSE
      DET=1.0
      MOUSE=ISIMEQ(20, JT, JT, D, UNIT, DET, SPACE)

      GO TO (5, 6, 6), MOUSE
      PRINT 100
6

```



```

100 FORMAT(20H      ERROR IN ISIMEQ )
      CALL EXIT
      FIND COEFFICIENTS
5     DO 13 M=1,MT
      DO 13 N=1,JT
          C(N,M)=0.0
      DO 13 J=1,JT
          C(N,M)=C(N,M)+D(N,J)*E(M,J)
13     CONTINUE
      EVALUATE POLYNOMIAL AT INPUT POINTS
      DO 18 M=1,MT
          R(M)=0.0
      DO 19 I=1,IT
          H(I,M)=0.0
      DO 7 J=1,JT
          H(I,M)=H(I,M)+C(J,M)*A(I,J)
7     CONTINUE
          COMPUTE ERROR
          R(M)=R(M)+(H(I,M)-DATA(I,M))**2
13     CONTINUE
          R(M)=SQRT(R(M))/QIT
13     CONTINUE
      RETURN
      END

```



```

SUBROUTINE LEGTR(K,INTVAL,A)
DIMENSION A(20,20)
C THIS SUBROUTINE COMPUTES A K BY K TRANSFORMATION MAR
C THIS SUBROUTINE COMPUTES A K BY K TRANSFORMATION MATRIX, A , TO
C CONVERT LEGENDRE POLYNOMIALS
C CONVERT LEGENDRE POLYNOMIAL COEFFICIENTS TO ORDINARY ONES.
C IF INTVAL=1 THE INTERVAL IS (0,1) . IF INTVAL=2 THE INTERVAL IS (-1,1)
DO 1 M=1,K
DO 1 N=1,K
      A(M,N)=0.0
1 CONTINUE
      A(1,1)=1.0
      GO TO (2,3),INTVAL
2      A(2,1)=-1.0
      A(2,2)=2.0
      GO TO 4
3      A(2,2)=1.0
4 DO 5 M=3,K
      B=M-1
      C=M-2
      D=2.0*M-3
      P=C/B
      GO TO (6,7),INTVAL
6      Q=D/B
      R=2.0*Q
      GO TO 8
7      Q=0.0
      R=D/B
      JT=M-1
      DO 5 J=1,JT
          A(M,J)=A(M,J)-P*A(M-2,J)-Q*A(M-1,J)
          A(M,J+1)=A(M,J+1)+R*A(M-1,J)
5 CONTINUE
      RETURN
      END

```



```

947 FUNCTION ISIMEQ ( DSM, NE, NC, A, B, DET, C )
948 SUBPROGRAM TO SOLVE SIMULTANEOUS LINEAR EQUATIONS
949 ARGUMENTS -
950 DSM DIMENSIONED SIZE OF COEFFICIENT MATRIX
951 NE ACTUAL NUMBER OF EQUATIONS FOR THIS CALL
952 NC NUMBER OF COLUMNS IN CONSTANT MATRIX
953 A COEFFICIENT MATRIX
954 B CONSTANT MATRIX
955 DET INPUT - SCALE FACTOR - OUTPUT - FACTOR TIMES
956 DETERMINANT VALUE OF COEFFICIENT MATRIX
957 C TEMPORARY STORAGE FOR SUBROUTINE
958 ISIMEQ RETURNS 1 IF OK, 2 IF OVRFLW, 3 IF SINGULAR
959 IF NC IS NEGATIVE, THE INVERSE OF THE COEFFICIENT MATRIX I
960 S REQUIRED, MATRIX B IS SET UP AS IDENTITY
961 LOGICAL DVO
962 INTEGER DSM,C,T,SUB1,SUB2,R,D
963 DIMENSION A(1), B(1), C(1)
964 INITIALIZE
965 N=NE
966 D=DSM
967 M=IABS(NC)
968 ISIMEQ=1
969 DVO=%FALSE%
970 DO 1 I=1,N
971 C(I)=1
972 IF(NC) 5,15,15
973 INVERSE REQUIRED
974 SUB2=0
975 DO 10 J=1,N
976 SUB1=SUB2
977 DO 6 I=1,N
978 SUB1=SUB1+1
979 B(SUB1)=0%0
980 SUB1=SUB2+J
981 B(SUB1)=1%0
982 SUB2=SUB2+D
983
984
985
986
987
988
989
990
991
992
993
994
995
996
997
998
999
1000

```



```

C 15 START MAIN LOOP
DO 1000 L=1,N
LP1=L+1
DO 40 I=L,N
PIVOT=0.0
SUB1=(L-1) * D + I
SUB2=SUB1
DO 20 J=L,N
IF ( ABS(PIVOT) .GE. ABS(A(SUB1))) GO TO 20
PIVOT = A(SUB1)
JB=J
20 SUB1 = SUB1 + D
C COMPUTE DETERMINANT
21 DET = DET * PIVOT
24 IF(PIVOT .EQ. 0.0) GO TO 2000
DO 25 J=L,N
A(SUB2) = A(SUB2) / PIVOT
SUB2 = SUB2 + D
25 IF (DVC) GO TO 35
SUB1 = I
DO 30 J=1,M
B(SUB1) = B(SUB1) / PIVOT
30 SUB1 = SUB1 + D
35 IF ( I .EQ. L) JP= JB
40 CONTINUE
C INTERCHANGE COLUMNS
100 IF (JP .EQ. L) GO TO 260
IF (DVC) GO TO 110
T= C(L)
C(L)=C(JP)
C(JP) = T
110 R = D * L - D
T = D * JP - D
DO 120 I=1,N
SUB1 = R + I
SUB2 = T + I

```

982
 983
 984
 985
 986
 987
 988
 989
 990
 991
 992
 993
 994
 995
 996
 997
 998
 999
 1000
 1001
 1002
 1003
 1004
 1005
 1006
 1007
 1008
 1009
 1010
 1011
 1012
 1013
 1014
 1015
 1016
 1017


```

1018 S = A(SUB1)
1019 A(SUB1) = A(SUB2)
1020 A(SUB2) = S
1021 DET = -DET
1022 REDUCE PIVOT COLUMN
1023 R = D * L - D
1024 DO 400 I=1,N
1025 IP = R + I
1026 PIVOT= A(IP)
1027 IF (I .EQ. L .OR. PIVOT .EQ. 0.0) GO TO 400
1028 SUB1 = L
1029 SUB2 = I
1030 DO 360 J = 1,N
1031 IF (J .LT. LP1) GO TO 300
1032 S = PIVOT * A(SUB1)
1033 A(SUB2) = A(SUB2) - S
1034 IF (ABS(A(SUB2)) .LT. ABS(3.0E-8 * S)) A(SUB2) = 0.0
1035 IF (DVC .OR. J .GT. M) GO TO 350
1036 B(SUB2) = B(SUB2) - PIVOT * B(SUB1)
1037 SUB1 = SUB1 + D
1038 SUB2 = SUB2 + D
1039 CONTINUE
1040 CONTINUE
1041 IF (DVC) GO TO 1500
1042 REARRANGE VARIABLES
1043 DO 1200 L = 1,N
1044 SUB1 = C(L)
1045 SUB2 = L
1046 DO 1200 J = 1,M
1047 A(SUB1) = B(SUB2)
1048 SUB1 = SUB1 + D
1049 SUB2 = SUB2 + D
1050 RETURN
1051 C SINGULAR COEFFICIENT MATRIX
1052 ISIMEQ= 3
1053 GO TO 1500

```


VALUES OF THE HYDROSTATIC FORCE AND MOMENT
AS A FUNCTION OF HEAVE AND ROLL DISPLACEMENTS
DISCUSSION OF PARAMETERS

Z/(R0*G*B**3) - NONDIMENSIONAL FORCE
M/(R0*G*B**4) - NONDIMENSIONAL MOMENT
Z(0)/(T/2) - NONDIMENSIONAL HEAVE DISPL.
FI - NONDIMENSIONAL ROLL DISPL.
RU - WATER DENSITY
G - GRAVITATIONAL ACCELERATION
B - BEAM OF THE MODEL
T - DRAFT OF THE MODEL

1 - HEAVE FORCE

ABS(FI)	1.00	.98	.96	.94	.92	0	.02	.04	.06	.08	1.00
.0	.79	.64	.49	.34	.17	0	-.18	-.37	-.52	-.71	-.90
.175	.77	.62	.48	.32	.16	-.01	-.18	-.36	-.54	-.73	-.92
.350	.72	.58	.44	.29	.14	-.03	-.20	-.35	-.53	-.70	-.87
.525	.64	.51	.38	.24	.10	-.06	-.20	-.34	-.50	-.63	-.76
.700	.52	.41	.30	.18	.05	-.07	-.19	-.30	-.42	-.52	-.62
.875	.36	.28	.20	.10	.00	-.07	-.15	-.23	-.31	-.37	-.43
1.000	.18	.14	.12	.04	-.02	-.06	-.11	-.16	-.20	-.24	-.26

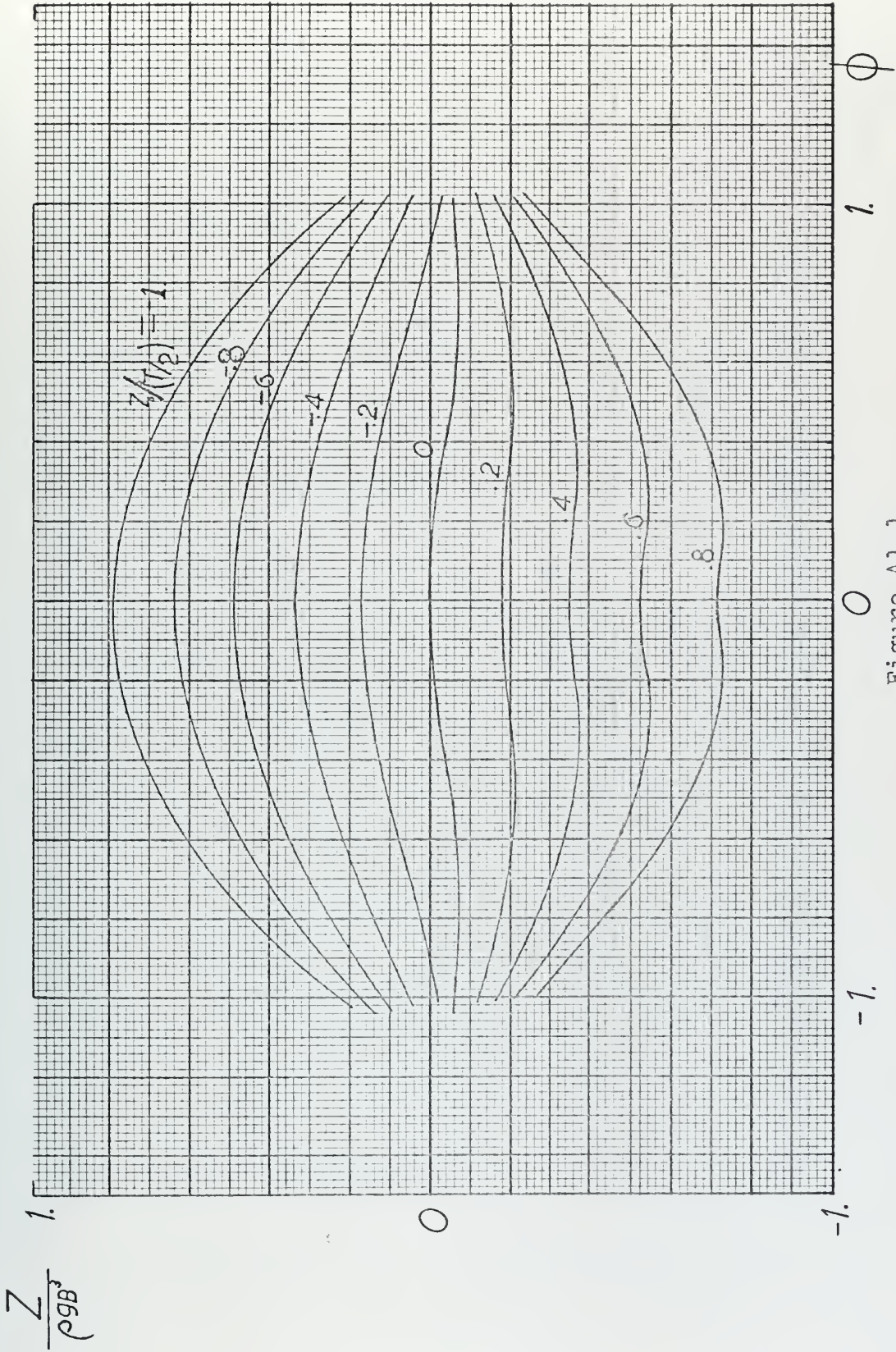


Figure A1.1

Hydrostatic force vs. angle of rotation and vertical position

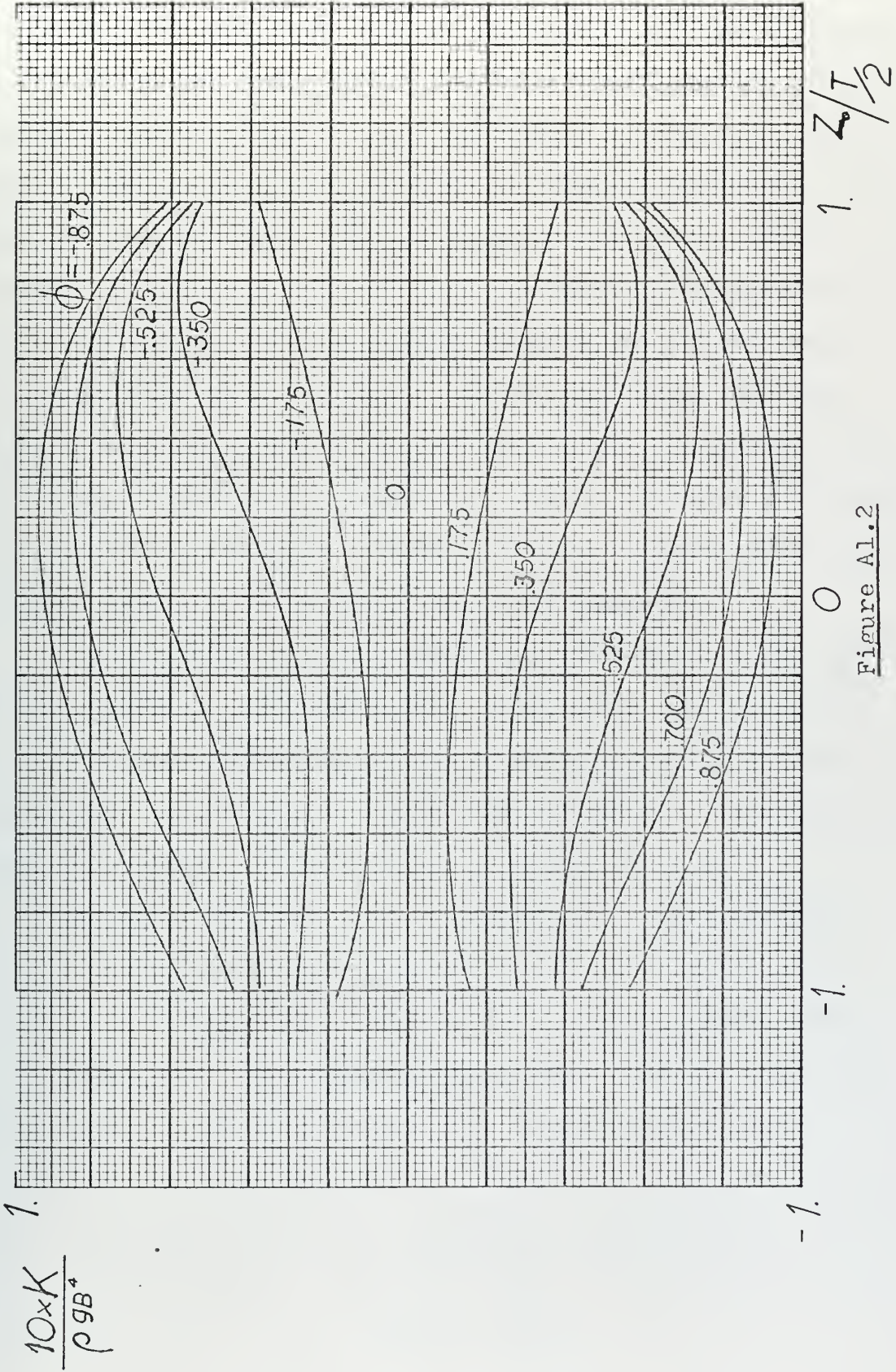


Figure AL.2

Hydrostatic moment vs. angle of rotation and vertical position

VALUES OF THE HYDROSTATIC COEFFICIENTS

Since the identification of the motion parameters in coupled heave and roll was not performed, it was necessary to calculate the hydrostatic coupling coefficients. To calculate only the coefficients Z_z , Z_{z2} , K_ϕ and $K_{\phi3}$ a much simpler method than resorting to the Kerwin polynomial representation and that will probably yield more accurate results can be used. The reason for such is that in computerized solutions using least squares, it is not usually possible to specify some a priori knowledge that one might have about the surface or the curve. For example in roll $K(\phi=0)$ and $K_{\phi2}|_{\phi=0}$ are zero due to port and starboard symmetry. Also $K_\phi|_{\phi=0} = -W*GM$. A least square method gave nonzero values for the first two coefficients.

Looking just for the second non zero term in the Taylor's expansion and knowing the first it's much easier to calculate manually the missing term. Using this procedure and being, for roll $K_0 = W*GM = -2.159 \text{ LB*FT}$, the value that will approximate the moment curve better will be :

$$\begin{array}{ll} \text{Between } \phi = \pm .175 & 1/3'K_{\phi3} = 6.814 \text{ LB*FT} \\ \phi = \pm .350 & = -1.254 \text{ LB*FT} \\ \phi = \pm .525 & = -3.007 \text{ LB*FT} \end{array}$$

Since the roll angle will go over .175 but does not reach .350 in the experiments the value chosen was:

$$1/3'K_{\phi3} = 6.814 \text{ LB*FT}$$

Similarly for heave the values found were

$$Z_z = -\rho*g*A_{lw} = -194.709 \text{ LB/FT}$$

$$1/2'Z_{z2} = -84.528 \text{ LB/FT}^2$$

APPENDIX 2

LINEAR HYDRODYNAMIC COEFFICIENTS

1 - DISCRIPTION OF THE COMPUTER PROGRAMS TO CALCULATE ADDED MASS, ADDED MOMENT OF INERTIA AND DAMPING COEFFICIENTS FOR TWO DIMENSIONAL CYLINDERS IN HEAVE, SWAY AND ROLL

2 - VALUES OF ADDED MASS, ADDED MOMENT OF INERTIA AND DAMPING COEFFICIENTS FOR A MODEL OF THE " MARINER " HULL FORM IN HEAVE AND ROLL

•

FORMAT(20A4)

The purpose of these cards is to provide titles for computed output. When titles are not desired blank should be provided.

```
FORMAT(3I6)
```

NOK - Total number of frequencies, at which the calculations will be performed.(MAX 50)

MAXB -Integer correspondent to the horizontal plane
maximum beam. Start the count at the keel.

NH - Number of depths at which the calculations will
formed.(MAX 10)

```
FORMAT(5F12.7)
```

CAY - Nondimensional frequencies - a/g

- radian frequency

a - half the beam for a floating cylinder

- half the draft for a submerged cylinder

g - gravitational acceleration

```
FORMAT(5F12.7)
```

DEP - Values of the depths of submergence, measured
the surface to the top of the cylinder. For a floa-
cylinder, NH 1 and DEP 0 .

```
FORMAT(2I6)
```

MD - Control integer - 1 for a submerged body

2 for a floating body

NON - Number of segments in which the right hand side half contour is divided (MAX 45)

CR - Positive vertical distance from the center of rotation to the free surface, for a floating cylinder or from the center of rotation to the intersection of the section contour with the plane of symmetry(on the side closer to the surface.

Card 9 - X(J),J 1, (NON 1) FORMAT(5F12.7)

X(J) - Horizontal offsets of the end points of the line segments, measured from the vertical planr of symmetry to the right hand side of the contour, starting at the keel. The last point is the intersection of the section contour either with the free surface (floating cylinder) or with the vertical plane of symmetry(submerged cylinder)

Card 10 - Y(J),J 1, (NON 1) FORMAT(5F12.7)

Y(J) - Negative vertical offsets of the end points of the line segments, measured from th free surface, for a floating cylinder or from the intersection of the contour with the vertical plane of symmetry, for a submerged cylinder, starting from the keel. Again only the right hand side of the contour will be considered.

Card 11 - MODE FORMAT(I6)

MODE - Control integer - 1 - heave
2 - sway
3 - roll

Card 12 - FORMAT(20A4)

Provides a title for the output of the program. If the title is not desired, a blank card should be provided.

If more than one mode is desired, additional pairs of cards 11 and 12 should be provided, accordingly.

Card 13 - MODE FORMAT(I6)

MODE - Control variable

0 - End of data

Negative integer - A new geometry follows
starting Card 5.

LISTINGS OF THE PROGRAMS

THE FOLLOWING PROGRAMS WERE TAKEN FROM
 NSRDC REPORT 3551
 " NUMERICAL CALCULATION OF ADDED MASS AND DAMPING
 COEFFICIENTS OF CYLINDERS OSCILLATING IN OR BELOW
 A FREE SURFACE "

BY
 J.W. BEDEL AND C.M. LEE

-121-

```

C      MAIN PROGRAM -- YFA4
      DIMENSION XA(46),YA(46)
      COMMON P1,HP1,WPI,TPI,MD,MCOL,DPH,CR,KAI,SUK,DEG,JERK,DRT,HBM,SG,N
      LUE,PDM,VOL,DCA,UN,OMEGA,CP,WVH,IO,DUG,IG,SLN(46),CES(46),XX(46),YY
      2(46),DEL(46),SNE(46),CSE(46),FR(46),BLDG(46,46),YLOG(46,46),CON(96
      3,2),CT(96,96),PSI1(46,46),PSI2(46,46),PRA(46),PRV(46),DEP(17)
      COMMON/GK/NOK,NUT,NON,TITLE(20),TITD(20),CAY(5),AMC(50),DFC(50),X
      1(46),Y(46)
      COMMON/FQ/K
      COMMON/TT/TEST
      COMMON/GKPH/TITD(20),DEF(50),LAD,LDT,TITA(20),TITV(20),PAK(50),PVR
      1(50),PAS(5),PVS(50),LPV
      COMMON/SHP/MXB,DUL(46)
      COMMON/MOL/F(5),D(5)
      1 FORMAT(12I6)
      2 FORMAT(5F16.7)
      3 FORMAT(20A4)
      4 FORMAT(77777777)
      1 SURFACE//6X,77H H/D = DEPTH TO TOP OF BODY / HALF DRAFT)
  
```



```

5  FORMAT(39H)
6  FORMAT(24H)
7  FORMAT(34H)
8  FORMAT(34H)
9  FORMAT(33H)
10 FORMAT(12H)
11 FORMAT(12H)
12 FORMAT(42H)
13 FORMAT(5F12.0)
14 FORMAT(39H)
29 FORMAT(29H)
34 FORMAT(6X,24H)
    ADFC = 1.0 FOR CAY = F4.2)
50  FORMAT(1H,12X,12H)
    1 CUEFFICIENT = F10.5)
55  FORMAT(1H,2X,30H)
    37  FORMAT(1H,5X,38H)
    42  FORMAT(1H,5X,42H)
    52  FORMAT(///,61H
    12 SURFACE)
54  FORMAT(2H MD =,13,4X,
    1,2H CR =,F10.4)
65  FORMAT(2H1,20A4)
66  FORMAT(///,2X,11H CAY VALUES//(5F12.7))
67  FORMAT(///,2X,13H INPUT VALUES///)
68  FORMAT(///,2X,7H DEPTHS//(5F12.7))
69  FORMAT(2H1,20A4//)
70  FORMAT(2X,///)
71  FORMAT(2H1 )
    PI=3.1415927
    HPI=65.4PI
    QPI=65.4HPI
    TPI=20.4PI
    F(1)=6652475561
    F(2)=665566681
    F(3)=667594245

```

TRANSFORMATION NOT DEFINED - STOP)

MATRIX IS SINGULAR)

HEAVING OSCILLATIONS, H/D = F10.5)

SWAYING OSCILLATIONS, H/D = F10.5)

ROLLING OSCILLATIONS ABOUT F10.5,8H H/D = F10.5)

CAY = F6.4,13H NUNWL = F8.4)

AMC = F10.5,9H CFC = F10.5,9H WVH = F10.5)

PRESSURES IN PHASE WITH ACCELERATION//)

PRESSURES IN PHASE WITH VELOCITY//)

DETERMINANT = E15.8)

DEPTH = F4.2,10H, APC = F8.4,8H DEGREE5/6X,20

BLAM = F10.5/17X,8H)AFT = F10.5/0X,19H)AREA

OF CYLINDRICAL CROSS SECTION//)

OF CYLINDRICAL CROSS SECTION//)

SURFACE CONDITION, NU VERY LARGE)

SEMI-SUBMERGED CYLINDERS OSCILLATING IN THE FRE

OH NCN =,13,4X,0H NOK =,13,4X


```

51 WRITE(6,52)
52 GO TO 55
53 WRITE(6,4)
54 WRITE(6,67) MD,MCN,NOK,CR
55 WRITE(6,54) (CAY(K),K=1,NOK)
56 WRITE(6,66) (DEP(J),J=1,NH)
57 WRITE(6,56) (XA(J),J=1,NUT)
58 WRITE(6,13) (YA(J),J=1,NUT)
59 WRITE(6,57) HOCAM,DRAFT,AREA
60 READ(5,1) MODE
61 IF(MODE) 40,43,41
62 READ(5,3)(TITLE(J),J=1,20)
63 DO 20 I5=1,NH
64 WRITE(6,69)(TITLE(J),J=1,20)
65 DPH=DEP(I5)
66 DO 78 J=1,NUT
67 X(J)=XA(J)
68 Y(J)=YA(J)
69 CALL SHAPED
70 CALL FIND
71 GO TO(21,22,23),MODE
72 WRITE(6,7)DPH
73 GO TO 24
74 WRITE(6,8)DPH
75 GO TO 27
76 WRITE(6,9)CK,DPH
77 WRITE(6,7A)
78 DO 20 K=1,NOK
79 UN=CAY(K)/PDM
80 CAGEA=SQRT(ABS(UN))
81 NLN=20*PI/ABS(CAY(K))
82 NN=CAY(K)/DRT
83 IF(UN)43,44,44

```



```
43 WRITE(6,42)
   GO TO 45
44 WRITE(6,10) CAY(K),WLN
45 CALL FREQ
   ID=ID
   GO TO(25,28),ID
28 WRITE(6,6)
   GO TO 48
25 WRITE(6,11)AMC(K),DFC(K),WVH
   WRITE(6,12)
   WRITE(6,13)(PRA(J),J=1,NCN)
   WRITE(6,14)
   WRITE(6,15)(PRV(J),J=1,NCN)
   WRITE(6,29)DUG
   PAK(K)=PRA(1)
   PVK(K)=PRV(1)
   PAS(K)=PRA(NCN)
   PVS(K)=PRV(NCN)
   IF(UM)46,46,47
46 DET(K)=0.5
   GO TO 48
47 DET(K)=DUG
48 WRITE(6,75)
24 CONTINUE
27 GO TO 38
40 WRITE(6,71)
   IF(MODE.LT.0) GO TO 55
   STOP
   END
```



```

SUBROUTINE SHAPED
CALCULATION OF GEOMETRIC PARAMETERS
COMMON PI,HPI,QPI,TPI,MD,MODE,DPH,CR,KAT,SUR,DEG,JERK,CRT,HBM,SG,N
10E,PDM,VOL,DEW,UN,OMEGA,CP,WVH,ID,DUG,IG,SEN(40),CES(46),XX(45),YY
2(45),DEL(45),SNE(45),CSE(45),FR(45),BLOG(45,45),YLOG(45,45),CON(90
3,1),CT(90,90),PSI1(45,45),PSI2(45,45),PRA(45),PRV(45)
COMMON/GR/NOX,NUT,NON,TITLE(20),TITU(20),CAY(50),AMC(50),DFC(50),X
1(46),Y(46)
COMMON/SHP/MAXB,DUL(45)
JERK=1
XAB=MAXB
GO TO (10,15),MD
15 D=X(KAB)
DPH=D*J
GO TO 55
10 D=.5*(Y(NUT)-Y(1))
53 DO 54 J=1,NUT
X(J)=X(J)/D
54 Y(J)=Y(J)/D
CRT=D
HBM=X(KAB)
CP=CR/D
IF(DPH) 24,24,21
21 DPH=DPH/D
DO 22 J=1,NUT
22 Y(J)=Y(J)-DPH
NUT=NON+1
24 DO 25 J=1,NON
XX(J)=.5*(X(J)+X(J+1))
YY(J)=.5*(Y(J)+Y(J+1))
25 DEL(J)=DUL(J)/D
SG=SING(MODE)
NDE=2*NON
GO TO (26,27,28),MODE
26 DO 29 J=1,NON
29 FR(J)=CSE(J)

```



```
33 PDM=1.0
DEW=1.0
30 TO (31,30),MD
30 VOL=HPI
30 TO 37
31 VOL=PI
30 TO 37
27 DO 32 J=1,NUN
32 FR(J)=-SNE(J)
30 TO 33
28 DO 34 J=1,NUN
34 FR(J)=(YY(J)-CP)*SNE(J)+XX(J)*CSE(J)
DEW=HBM
PDM=1.0
30 TO (36,35),MD
35 VOL=QPI
30 TO 37
30 VOL=HPI
37 RETURN
END
```



```

SU2960U1955 0954
C CALCULATION OF THE NORMAL DERIVATIVES OF THE LOGARITHMIC SINGULARITIES
COMMON PI,HPI,QPI,TPI,MD,MODE,DPH,CR,RAT,SUR,DEG,JERK,DRT,HBM,SG,NFIND0020
10E,PDM,VOL,DEW,UN,CMEGA,CP,WVH,ID,DUG,IG,SEN(46),CES(46),XX(45),YYFIND0030
2(45),DEL(45),SNE(45),CSE(45),FR(45),BLOG(45,45),YLOG(45,45),CON(90FIND0040
3,1),CT(90,90),PSI1(45,45),PSI2(45,45),PRA(45),PRV(45)
COMMON/GK/NOK,NUT,NON,TITLE(20),TITO(20),CAY(50),AMC(50),DFC(50),X
1(46),Y(46)
COMMON/SHP/MAXB,DUL(45)
DO 1 I=1,NON
  XM1=XX(I)-X(1)
  YM1=YY(I)-Y(1)
  XP1=XX(I)+X(1)
  YP1=YY(I)+Y(1)
  FPR1=.5*ALOG(XM1**2+YM1**2)
  FPL1=.5*ALOG(XP1**2+YM1**2)
  FCR1=.5*ALOG(XM1**2+YP1**2)
  FCL1=.5*ALOG(XP1**2+YP1**2)
  APR1=ATAN2(YM1,XM1)
  APL1=ATAN2(YM1,XP1)
  ACR1=ATAN2(YP1,XM1)
  ACL1=ATAN2(YP1,XP1)
  DO 1 J=1,NON
    XM2=XX(I)-X(J+1)
    YM2=YY(I)-Y(J+1)
    XP2=XX(I)+X(J+1)
    YP2=YY(I)+Y(J+1)
    FPR2=.5*ALOG(XM2**2+YM2**2)
    FPL2=.5*ALOG(XP2**2+YM2**2)
    FCR2=.5*ALOG(XM2**2+YP2**2)
    FCL2=.5*ALOG(XP2**2+YP2**2)
    APR2=ATAN2(YM2,XM2)
    APL2=ATAN2(YM2,XP2)
    ACR2=ATAN2(YP2,XM2)
    ACL2=ATAN2(YP2,XP2)
    SIMJ=SNE(I)*CSE(J)-SNE(J)*CSE(I)
  
```

```

FIND0070
FIND0080
FIND0090
FIND0100
FIND0110
FIND0120
FIND0130
FIND0140
FIND0150
FIND0160
FIND0210
FIND0220
FIND0230
FIND0240
FIND0250
FIND0260
FIND0270
FIND0280
FIND0290
FIND0300
FIND0310
FIND0320
FIND0330
FIND0340

```



```

CIMJ=CSE(I)*CSE(J)+SNE(I)*SNE(J)
SIPJ=SNE(I)*CSE(J)+SNE(J)*CSE(I)
CIPJ=CSE(I)*CSE(J)-SNE(I)*SNE(J)
OPNR=SIMJ*(FPR1-FPR2)+CIMJ*(APR1-APR2)
PPR=CSE(J)*(XMI*FPR1-YM1-YM2*FPR2-XM1-XM2*FPR2+YM2*APR2+XM2)+SNE(J)*(YFIND0350
1MI*FPR1+XMI*APR1-YM1-YM2*FPR2-XM1-XM2*APR2+YM2)
OPNL=SIPJ*(FPL2-FPL1)+CIPJ*(APL2-APL1)
PPL=CSE(J)*(XP2*FPL2-YM2*APL2-XP2-XP1*FPL1+YM1*APL1)+SNE(J)*(YFIND0420
1MI*FPL1+XP1*APL1+YM2-YM2*FPL2-XP2*APL2-YM1)
DCNR=SIPJ*(FCR1-FCR2)+CIPJ*(ACR1-ACR2)
PCR=CSE(J)*(XMI*FCR1-YP1-YP1*FCR1-XM1-XM2*FCR2+YP2*ACR2+XM2)+SNE(J)*(YFIND0450
1P2*FCR2+XM2*ACR2+YP1-YP1*FCR1-XM1-XM2*ACR1-YP2)
DCNL=SIMJ*(FCL2-FCL1)+CIMJ*(ACL2-ACL1)
PCL=CSE(J)*(XP2*FCL2-YP2*ACL2-XP2-XP1*FCL1+YP1*ACL1+XP1)+SNE(J)*(YFIND0480
1P2*FCL2+XP2*ACL2-YP2-YP1*FCL1-XP1*ACL1+YP1)
BLUG(I,J)=DPNR+SG*DPNL-DCNR-SG*DCNL
YLOG(I,J)=PPR+SG*PPL-PCR-SG*PCL
IF(J-NCN)2,1,1
2 XMI=XM2
YM1=YM2
XP1=XP2
YP1=YP2
FPR1=FPR2
FPL1=FPL2
FCR1=FCR2
FCL1=FCL2
APR1=APR2
APL1=APL2
ACR1=ACR2
ACL1=ACL2
1 CONTINUE
RETURN
END

```

FIND0350
 FIND0360
 FIND0370
 FIND0380
 FIND0390
 FIND0400
 FIND0410
 FIND0420
 FIND0430
 FIND0440
 FIND0450
 FIND0460
 FIND0470
 FIND0480
 FIND0490
 FIND0500
 FIND0510
 FIND0520
 FIND0530
 FIND0540
 FIND0550
 FIND0560
 FIND0570
 FIND0580
 FIND0590
 FIND0600
 FIND0610
 FIND0620
 FIND0630
 FIND0640
 FIND0650
 FIND0660
 FIND0670


```

SUBROUTINE DAVID(X,Y,E,C,S,RA,RB,CIN,SON)
COMPUTATION OF EXPONENTIAL INTEGRAL WITH COMPLEX ARGUMENT
COMMON/MOD/F(5),D(5)
Q=3.1415927
AT=ATAN2(X,Y)
ARG=AT-.5*Q
E=EXP(-Y)
C=COS(X)
S=SIN(X)
R=X**2+Y**2
AL=Q*.5*ALOG(R)
A=-Y
B=-X
IF(A.GE.0.0) GO TO 78
IF(B.EQ.0.0) GO TO 79
78 IF(R.GE.100.) GO TO 10
79 TEST=0.0(001
IF(R.LT.1.0) GO TO 5
TEST=0.1*TEST
IF(R.LT.2.0) GO TO 5
TEST=0.1*TEST
IF(R.LT.4.0) GO TO 5
TEST=0.1*TEST
5 CONTINUE
SUMC=Q.57721566+AL+Y
SUMS=AT+X
TC=Y
TS=X
DO 1 K=1,500
TU=TC
COX=K
CAY=K+1
FACT=COX/CAY**2
TC=FACT*(Y*TC-X*TS)
TS=FACT*(Y*TS+X*TU)
SUMC=SUMC+TC

```



```
SUMS=SUMS+TS
IF(K.GE.500) GO TO 3
IF((ABS(TC)+ABS(TS)).GT.TEST) GO TO 1
3 CIN=E*(C*SUMC+S*SUMS)
SUN=E*(S*SUMC-C*SUMS)
GO TO 4
1 CONTINUE
10 G1=0.
G2=0.
DO 20 I=1,5
DEN=(-Y+D(I))**2+X**2
GA=F(I)*(-Y+D(I))/DEN
GB=F(I)*(-X)/DEN
G1=G1+GA
G2=G2+GB
20 CIN=E*Q*S-G1
SUN=-(E*Q*C+G2)
4 RA=AL-CIN
RB=ARG+SUN
RETURN
END
```



```
C      FUNCTION SING(N)  
C      ASSIGNMENT OF POSITIVE OR NEGATIVE SIGNAL, DEPENDING ON THE MODE OF  
C      MOTION  
      IF(1-N)2,1,1  
          1 SING=1.  
          GO TO 77  
          2 SING=-1.  
      77 RETURN  
      END
```



```

SUBROUTINE FREQ
CALCULATION OF THE SOURCE STRENGTH ON THE CONTOUR SEGMENTS
COMMON PI,HPI,QPI,TPI,MD,MODE,DPH,CR,RAI,SUK,DEG,JERK,DRT,HBM,SG,N
10E,PDM,VOL,DEW,UN,OMEGA,CP,WVH,ID,DUG,IG,SEN(46),CES(46),XX(45),YY
2(45),DEL(45),SNE(45),CSE(45),FR(45),BLOG(45,45),YLOG(45,45),CON(90)
3,1),CT(90,90),PSI1(45,45),PSI2(45,45),PRA(45),PRV(45)
COMMON/GR/NOK,NUT,NON,TITLE(20),TITO(20),CAY(50),AMC(50),DFC(50),X
.1(46),Y(46)
COMMON/FQ/K
COMMON/IT/TEST
IF(UN)8,9,10
8 DO 11 I=1,NUN
DO 11 J=1,NON
CT(I,J)=BLOG(I,J)
11 PSI1(I,J)=YLOG(I,J)
GO TO 14
9 DO 12 I=1,NUN
XM1=XX(I)-X(1)
XP1=XX(I)+X(1)
YP1=YY(I)+Y(1)
FCR1=.5*ALOG(XM1**2+YP1**2)
FCL1=.5*ALOG(XP1**2+YP1**2)
ACR1=ATAN2(YP1,XM1)
ACL1=ATAN2(YP1,XP1)
DO 12 J=1,NON
XM2=XX(I)-X(J+1)
XP2=XX(I)+X(J+1)
YP2=YY(I)+Y(J+1)
FCR2=.5*ALOG(XM2**2+YP2**2)
FCL2=.5*ALOG(XP2**2+YP2**2)
ACR2=ATAN2(YP2,XM2)
ACL2=ATAN2(YP2,XP2)
SIMJ=SNE(I)*CSE(J)-SNE(J)*CSE(I)
CIMJ=CSE(I)*CSE(J)+SNE(I)*SNE(J)
SIPJ=SNE(I)*CSE(J)+SNE(J)*CSE(I)
CIPJ=CSE(I)*CSE(J)-SNE(I)*SNE(J)

```



```

DCNR=SI*PJ*(FCR1-FCR2)+CI*PJ*(ACR1-ACR2)
PCR=CSE(J)*(XM1*FCR1-YP1*ACR1-XM1-XM2*FCR2+YP2*ACR2+XM2)+SNE(J)*(Y
1P2*FCR2+XM2*ACR2+YP1-YP1*FCR1-XM1*ACR1-YP2)
DCNL=SI*PJ*(FCL2-FCL1)+CI*PJ*(ACL2-ACL1)
PCL=CSE(J)*(XP2*FCL2-YP2*ACL2-XP2-XP1*FCL1-XP1*ACL1+YP1)
1P2*FCL2+XP2*ACL2-YP2-YP1*FCL1-XP1*ACL1+YP1)
CT(I,J)=BLOG(I,J)+2.0*(DCNR+SG*PCL)
PSI1(I,J)=YLOG(I,J)+2.0*(PCR+SG*PCL)
1F(J-NON)13,12,12
13 XM1=XM2
XP1=XP2
YP1=YP2
FCR1=FCR2
FCL1=FCL2
ACR1=ACR2
ACL1=ACL2
12 CONTINUE
14 DO 15 I=1,NUN
15 CON(I,1)=FR(I)
CALL MATINV(CT,NON,CON,1,DIG,ID)
GO TO(16,6),ID
16 DO 17 I=1,NON
PRA(I)=0.0
PRV(I)=0.0
DO 17 J=1,NON
17 PRA(I)=PRA(I)-CJN(J,1)*PSI1(I,J)
AMC(K)=0.0
DFC(K)=0.0
NVH=0.0
DO 18 I=1,NON
18 AMC(K)=AMC(K)+PRA(I)*DEL(I)*FR(I)
AMC(K)=2.0*AMC(K)/VJL
GO TO 6
10 DO 1 I=1,NON
NI=NUN+I
CON(I,1)=0.0

```



```

CCN(NI,I)=OMEGA*FR(I)
XR1=UN*(XX(I)-X(1))
YR1=-UN*(YY(I)+Y(1))
XL1=UN*(XX(I)+X(1))
YL1=YR1
CALL DAVID(XR1,YR1,EJ1,CXR1,SR1,RBR1,CR1,SL1)
CALL DAVID(XL1,YL1,EJ1,CXL1,SL1,RBL1,CL1,SL1)
DO 1 J=1,NON
  NJ=NJN+J
  XR2=UN*(XX(I)-X(J+1))
  YR2=-UN*(YY(I)+Y(J+1))
  XL2=UN*(XX(I)+X(J+1))
  YL2=YR2
  CALL DAVID(XR2,YR2,EJ2,CXR2,SR2,RBR2,CR2,SL2)
  CALL DAVID(XL2,YL2,EJ2,CXL2,SL2,RBL2,CL2,SL2)
  SIPJ=SNE(I)*CSE(J)+SNE(J)*CSE(I)
  CIPJ=CSE(I)*CSE(J)-SNE(I)*SNE(J)
  SIMJ=SNE(I)*CSE(J)-SNE(J)*CSE(I)
  CIMJ=CSE(I)*CSE(J)+SNE(I)*SNE(J)
25 CT(I,J)=BLOG(I,J)+2.*(SIPJ*(CR1-CR2)-CIPJ*(SR1-SR2)-SG*(SIMJ*(CL1-
  CL2)-CIMJ*(SL1-SL2)))
  PSI1(I,J)=YLOG(I,J)+2./UN*(SNE(J)*(KAR1-RAK2)+CSE(J)*(RBR1-RBR2)+S
  IG*(SNE(J)*(RAL1-RAL2)+CSE(J)*(RBL2-RBL1)))
  CT(NI,NJ)=CT(I,J)
  CT(1,NJ)=TPI*(EJ2*(SXR2*CIPJ-CXR2*SIPJ)-EJ1*(SXR1*CIPJ-CXR1*SIPJ)-
  1SG*(EJ2*(SXL2*CIMJ-CXL2*SIMJ)-EJ1*(SXL1*CIMJ-CXL1*SIMJ)))
  PSI2(I,J)=TPI/UN*(SXR1*CSE(J)-CXR1*SNE(J)-EJ2*(SXR2*CSE(J)-C
  1XR2*SNE(J))-SG*(EJ1*(SXL1*CSE(J)+CXL1*SNE(J)-EJ2*(SXL2*CSE(J)+CXL
  2*SNE(J))))
  CT(NI,J)=-CT(I,NJ)
  IF(J-NON)7,1,1
7  XR1=XR2
  YR1=YR2
  XL1=XL2
  YL1=YL2
  EJ1=EJ2

```

FREQ


```

CR1=CR2
SR1=SR2
CL1=CL2
SL1=SL2
RAR1=RAR2
RBR1=RBR2
RAL1=RAL2
RBL1=RBL2
CXR1=CXR2
SXR1=SXR2
CXL1=CXL2
SXL1=SXL2
1 CONTINUE
CALL MATINV(CT,NCE,CCN,1,DOG,ID)
GJ TJ(2,0),ID
2 DO 3 I=1,NUN
  PRA(I)=0.
  PRV(I)=0.
  DO 4 J=1,NUN
    NJ=NGN+J
    PRA(I)=PRA(I)+CUN(J,I)*PSI2(I,J)-CCN(NJ,I)*PSI1(I,J)
    PRV(I)=PRV(I)+CUN(J,I)*PSI1(I,J)+CCN(NJ,I)*PSI2(I,J)
    PRA(I)=CMEGA*PRA(I)
    PRV(I)=CMEGA*PRV(I)
  3   AMC(K)=0.
      DFC(K)=0.
      DO 5 I=1,NUN
        AMC(K)=AMC(K)+PRA(I)*DEL(I)*FR(I)
        DFC(K)=DFC(K)+PRV(I)*DEL(I)*FR(I)
        AMC(K)=2.0*AMC(K)
        DFC(K)=2.0*DFC(K)
      WDH=CMEGA*SQRT(ABS(DFC(K)))/DEW
      AMC(K)=AMC(K)/(UN*VOL)
      DFC(K)=DFC(K)/(UN*VOL)
    6 RETURN
  END

```



```

C SUBROUTINE MATINV(A,N1,B,M1,DETERM,ID)
C PIVOT METHOD
C   MATRIX INVERSION WITH ACCOMPANYING SOLUTION OF SIMUL. EQ.
C PIVOT METHOD
C   FORTRAN IV SINGLE PRECISION WITH ADJUSTABLE DIMENSION
C   FEBRUARY 1966 S GOOD DAVID TAYLOR MODEL BASIN AM MAT4
C   WHERE CALLING PROGRAM MUST INCLUDE
C   DIMENSION A( ), B( ), INDX( )
C   N IS THE ORDER OF A
C   M IS THE NUMBER OF COLUMN VECTORS IN B(MAY BE 0)
C   DETERM WILL CONTAIN DETERMINANT ON EXIT
C   ID WILL BE SET BY ROUTINE TO 2 IF MATRIX A IS SINGULAR
C   1 IF INVERSION WAS SUCCESSFUL
C   A THE INPUT MATRIX WILL BE REPLACED BY A INVERSE
C   B THE COLUMN VECTORS WILL BE REPLACED BY CORRESPONDING
C     SOLUTION VECTORS
C   INDEX WORKING STORAGE ARRAY
C   IF IT IS DESIRED TO SCALE THE DETERMINANT CARD MAY BE
C   DELETED AND DETERM PRESET BEFORE ENTERING THE ROUTINE
C
C   DIMENSION A(90,90),B(90,1),INDEX(90,3)
C   EQUIVALENCE (IROW,JROW), (ICOL,JCOL), (AMAX,T, SWAP)
C   INITIALIZATION
C
C   N=N1
C   M=M1
C   DETERM = 100
C   DO 20 J=1,N
C   INDEX(J,3) = 0
C   DO 550 I=1,N
C
C   SEARCH FOR PIVOT ELEMENT
C
C   AMAX = 0.0
C   DO 105 J=1,N

```

MAT400001
 MAT400002
 MAT400003
 MAT400004
 MAT400005
 MAT400006
 MAT400007
 MAT400008
 MAT400009
 MAT400010
 MAT400011
 MAT400012
 MAT400013
 MAT400014
 MAT400015
 MAT400016
 MAT400017
 MAT400018
 MAT400019
 MAT400020

 MAT400022
 MAT400023
 MAT400024
 MAT400026
 MAT400027
 MAT400028
 MAT400029
 MAT400030
 MAT400031
 MAT400032
 MAT400033
 MAT400034
 MAT400035
 MAT400036
 MAT400037


```

      IF(INDEX(J,3)-1) 50, 105, 50
      DO 100 K=1,N
      IF(INDEX(K,3)-1) 80, 100, 710
      30 IF (      AMAX -ABS (A(J,K))) 85, 100, 100
      85 IROW=J
      ICOLUM =K
      AMAX = ABS (A(J,K))
      100 CONTINUE
      105 CONTINUE
      INDEX(ICOLUM,3) = INDEX(ICOLUM,3) +1
      INDEX(I,1)=IROW
      INDEX(I,2)=ICOLUM
      C
      C INTERCHANGE ROWS TO PUT PIVOT ELEMENT ON DIAGONAL
      C
      IF (IROW-ICOLUM) 140, 310, 140
      140 DETERM=-DETERM
      DO 200 L=1,N
      SWAP=A(IROW,L)
      A(IROW,L)=A(ICOLUM,L)
      200 A(ICOLUM,L)=SWAP
      IF(M) 310, 310, 210
      210 DO 250 L=1, M
      SWAP=B(IROW,L)
      B(IROW,L)=B(ICOLUM,L)
      250 B(ICOLUM,L)=SWAP
      C
      C DIVIDE PIVOT ROW BY PIVOT ELEMENT
      C
      310 PIVOT =A(ICOLUM,ICOLUM)
      DETERM=DETERM*PIVOT
      330 A(ICOLUM,ICOLUM)=100
      DO 350 L=1,N
      350 A(ICOLUM,L)=A(ICOLUM,L)/PIVOT
      IF(M) 380, 380, 360
      360 DO 370 L=1,M

```

MAT40038
 MAT40039
 MAT40040
 MAT40041
 MAT40042
 MAT40043
 MAT40044
 MAT40045
 MAT40046
 MAT40047
 MAT40048
 MAT40049
 MAT40050
 MAT40051
 MAT40052
 MAT40053
 MAT40054
 MAT40055
 MAT40056
 MAT40057
 MAT40058
 MAT40059
 MAT40060
 MAT40061
 MAT40062
 MAT40063
 MAT40064
 MAT40065
 MAT40066
 MAT40067
 MAT40068
 MAT40069
 MAT40070
 MAT40071
 MAT40072
 MAT40073


```

370 B(ICOLU,M,L)=B(ICOLU,M,L)/PIVOT
C
C      REDUCE NON-PIVOT ROWS
C
380 DO 550 L1=1,N
    IF(L1-ICOLU) 400, 550, 400
400 T=A(L1,ICOLU)
    A(L1,ICOLU)=0.0
    DO 450 L=1,N
450 A(L1,L)=A(L1,L)-A(ICOLU,L)*T
    IF(M) 550, 550, 400
450 DO 500 L=1,M
500 B(L1,L)=B(L1,L)-B(ICOLU,L)*T
550 CONTINUE
C
C      INTERCHANGE COLUMNS
C
DO 710 I=1,N
L=N+1-I
    IF (INDEX(L,1)-INDEX(L,2)) 630, 710, 630
630 JROW=INDEX(L,1)
    JCOLUM=INDEX(L,2)
    DO 705 K=1,N
    SWAP=A(K,JROW)
    A(K,JROW)=A(K,JCOLUM)
    A(K,JCOLUM)=SWAP
705 CONTINUE
710 CONTINUE
DO 730 K = 1,N
    IF(INDEX(K,3) -1) 715,720,715
720 CONTINUE
730 CONTINUE
    ID = 1
810 RETURN
715 ID = 2
    GO TO 810

```

MAT40074
MAT40075
MAT40076
MAT40077
MAT40078
MAT40079
MAT40080
MAT40081
MAT40082
MAT40083
MAT40084
MAT40085
MAT40086
MAT40087
MAT40088
MAT40089
MAT40090
MAT40091
MAT40092
MAT40093
MAT40094
MAT40095
MAT40096
MAT40097
MAT40098
MAT40099
MAT40100
MAT40101
MAT40102
MAT40103
MAT40104
MAT40105
MAT40106
MAT40107
MAT40108
MAT40109

MA T4C 110

END

ADDED MASS, ADDED MOMENT OF INERTIA AND DAMPING COEFFICIENT

DISCRIPTION OF PARAMETERS

ST - STATION SPACING = .275 FT
 B/2 - HALF THE BEAM AT THE CORRESPONDENT STATION (FT)
 AMC - NONDIMENSIONAL LOCAL ADDED MASS OR ADDED MOMENT OF INERTIA
 AMC = $A(3,3)/((R0*(PI/2))*(B/2)**2)$ FOR HEAVE
 AMC = $A(3,3)/((R0*(PI/4))*(B/2)**4)$ FOR ROLL
 DFC - NONDIMENSIONAL LOCAL DAMPING COEFFICIENT
 DFC = $B(3,3)/((R0*(PI/2))*(B/2)**2*OMEGA)$ FOR HEAVE
 DFC = $B(4,4)/((R0*(PI/4))*(B/2)**4*OMEGA)$ FOR ROLL

ST	B/2	AMC	DFC	A(3,3)	B(3,3)
1/2	.0269	2.0343	.5051	.0028	.0101
1	.0487	1.0113	.8234	.0073	.0478
2	.0624	.5084	.9033	.0181	.2473
3	.0658	.5069	.9257	.0470	.0230
4	.0517	.5544	.8350	.0972	1.0985
5	.02934	.0324	.7373	.1658	1.05539
6	.03420	.06743	.6463	.2402	1.03513
7	.03742	.7342	.5580	.3131	1.09129
8	.03913	.8105	.4749	.3784	1.07818
9	.03956	.8824	.4143	.4210	1.05890
10	.03956	.9129	.3934	.4356	1.05088
11	.03956	.9129	.3934	.4350	1.03188
12	.03956	.9002	.3984	.4342	1.02280
13	.03956	.8458	.4420	.4336	1.00951
14	.03524	.7322	.5483	.3434	2.00009
15	.03794	.6447	.6820	.2826	2.04034
16	.03482	.6162	.8101	.2276	2.04044

17 02891 00221 09432 01584 109301
18 02023 00529 101211 00814 101213
19 00990 06201 104349 00244 03499
19 1/2 00261 306232 403805 00075 00815

SUM(A(3,3)) = 405105 LB*SEC**2/FT**2
SUM(B(3,3)) = 2701175 LB*SEC/FT**2

A(3,3) = 102404 LB*SEC**2/FT
B(3,3) = 704573 LB*SEC/FT

ST	2 -	ROLL (OMEGA	=	4.145	DFC	A(4,4)	SEC(-1)	B(4,4)
1/2	B/2	6445	1500		305	6050	0051	0010	
1	0269	573	9870		29	1870	0049	0020	
2	0437	22	3790		1	2852	0037	0009	
3	0124	1	6005		0	1154	0021	0005	
4	0160	02317	01297		0	0081	0006	0001	
5	02934	00074	00074		0	0001	0001	0000	
6	03420	00370	00370		0	0020	0008	0002	
7	03742	00674	00674		0	0023	0020	0003	
8	03915	01040	01040		0	0010	0037	0001	
9	03958	01487	01487		0	0000	0050	0000	
10	03956	01747	01747		0	0002	0065	0000	
11	03958	01747	01747		0	0002	0065	0000	
12	03958	01602	01602		0	0001	0063	0000	
13	03958	01240	01240		0	0004	0046	0001	
14	03924	00703	00703		0	0050	0027	0007	
15	03794	00857	00857		0	0121	0027	0010	
16	03432	01288	01288		0	0133	0029	0012	
17	02891	01754	01754		0	0052	0019	0002	
18	02023	07017	07017		0	0051	0010	0001	
19	00998	22	3258		0	6641	0034	0004	
19 1/2	00261	127	5578		1	5351	0001	0000	

SUM(A(4,4)) = 00039 LB*SEC**2
SUM(B(4,4)) = 00080 LB*SEC

A(4,4) = 00170 LB*SEC**2*FT
B(4,4) = 00022 LB*SEC*FT

ST	B/2	ROLL (OMEGA	=	0.029	DFC	A(4,4)	SEC(-1)	B(4,4)
1/2	0.0269	5819.8489	5.134	764			0.046		0.0339
1	0.0407	499.357	44.0	6100			0.043		0.0313
2	0.1024	16.5370	14.0	6124			0.031		0.0203
3	0.1658	1.4157		9331			0.016		0.0089
4	0.2317	0.1010		6433			0.004		0.0016
5	0.2934	0.0074		0.011			0.001		0.0001
6	0.3427	0.0291		0.056			0.006		0.0017
7	0.3742	0.0392		0.053			0.018		0.0015
8	0.3915	0.1024		0.002			0.037		0.0001
9	0.3958	0.1508		0.035			0.056		0.0011
10	0.3958	0.1759		0.082			0.066		0.0025
11	0.3958	0.1759		0.082			0.066		0.0025
12	0.3958	0.1699		0.069			0.063		0.0021
13	0.3958	0.1249		0.004			0.047		0.0001
14	0.3924	0.0015		0.117			0.022		0.0035
15	0.3794	0.0528		0.513			0.017		0.0134
16	0.3432	0.1033		0.752			0.091		0.0134
17	0.2891	0.1730		0.472			0.018		0.0042
18	0.2023	0.6845		0.188			0.0427		0.0004
19	0.0998	21.06162		7.3803			0.0033		0.0093
19 1/2	0.0261	133.5900		18.0078			0.0001		0.0001

SUM(A(4,4)) = 0.1171 LB*SEC*12
SUM(B(4,4)) = 0.1522 LB*SEC

A(4,4) = 0.0322 LB*SEC*2*FT
B(4,4) = 0.0363 LB*SEC*FT

APPENDIX 3

NON-LINEAR MOTION PARAMETERS

1 - DETERMINATION OF HYDRODYNAMIC COEFFICIENTS USING EMPIRICAL METHODS

1 - ADDED MASS AND ADDED MOMENT OF INERTIA

Using Ref. 20 p. 61 to 63 - Heave

$$\Delta m = \xi * \rho * \pi/2 \int_0^L \xi_z \cdot y^2 (dx) \quad (A3.1)$$

where ρ - water density 1.94 LB*SEC²/FT⁴

L - length of the model 5.5 FT

ξ_z - coefficient from fig. 17 ≈ 1 .

y - contour of the waterline, approximated by a parabola of the form

$$y = B/2 \left[1 - \frac{x}{L/2} \right]^n \quad (A3.2)$$

B -- beam of the model .79 FT

n = $C_w / (1 - C_w)$

C_w - waterplane coefficient = $A_{wl} / B * L = .724$

ξ_1 - coefficient, calculated according to

$$\xi_1 = 1 / [1 + .8 * B / L] = .895 \quad (A3.3)$$

the following form may be obtained

$$\Delta m = .772 * \xi_1 * \rho * L * B^2 * C_w^2 / (1 + C_w) \quad (A3.4)$$

Plugging in the values

$$\Delta m = 1.41 \text{ LB*SEC}^2/\text{FT}$$

Being the mass of the vehicle = m 1.43 LB*SEC²/FT

this result may be checked by measuring the natural period

of heaving motions and using results from the hydrostatic calculations $Z_z = - *g*A_{w1} = -194.05 \text{ LB/FT}$

$$\omega_n = \left[\frac{-Z_z}{m + \Delta m} \right]^{\frac{1}{2}} = 8.29 \text{ SEC}(-1)$$

The value found for the natural undamped frequency was 8.038 (SEC-1).

Similarly for roll, the added moment of inertia is expressed as a percentage of the moment of inertia. This is found, from data reffering to full scale vehicles, according to the formula

$$\begin{aligned} I_{xx} &= W/g*((B^2*C_w^2)/11.4*C_b + D^2/12) \text{ TON*SEC}^2 \quad (A3.5) \\ &= .02855 \text{ LBSEC}^2 \end{aligned}$$

From fig. 24 of the above refference

$$\Delta I_{xx} = .286 * I_{xx}$$

Therefore

$$I_{xx} + \Delta I_{xx} = .03665 \text{ LB*SEC}^2$$

From the hydrostatic calculations

$$K_\phi = - W*GM = -2.159 \text{ LB*FT}$$

The resulting natural frequency is then

$$\omega_n = 7.7 \text{ SEC}(-1)$$

while the value found in the measurements was

$$\omega_n = 4.14 \text{ SEC}(-1)$$

The values of added mass and of added moment of inertia, calculated in Appendix 2 were :

$$A(3,3) = 1.24 \text{ LB*SEC}^2/\text{FT}$$

$$A(4,4) = .0176 \text{ LB*SEC}^2*\text{FT}$$

The value of roll added moment of inertia is seen to be quite different the results of both methods are compared.

2 - DAMPING COEFFICIENT

For heave the following expression, taken from page 153 of the above reference was used :

$$Z_w = -1/2 \rho \omega^2 \pi / \lambda (L.B.C_w)^2 \chi^2 (T/\lambda) \cdot (U.L/\lambda) \quad (A3.6)$$

where the parameters are as before and also :

λ - wave length = 3.11 FT for $\omega = 8.04 \text{ SEC}^{-1}$

χ^2 - coefficient taken from (20) fig. 52 = .460

T - draft of the vehicle = .28 FT

$U.L$ - coefficient taken from (20) fig. 53 = .200

For the values above : $Z_w = -2.336 \text{ LB*SEC/FT}$

while when calculated by the methods of Appendix 2

$Z_w = -7.4543 \text{ LB*SEC/FT}$ which is three times the value presented above.

For roll, the calculation of linear and non-linear damping coefficients were carried simultaneously, for each transverse section of the vehicle, using Equations 21, 22, 23 or 24 of page 148 of the above reference, depending on the shape of the section. The value of the damping for the whole vehicle was obtained by summing the sectional results

over the length of the vehicle in a " strip theory " way.

Therefore, using the following equations the damping is calculated.

$$K_{Di} = \frac{1}{2} \rho b_i^4 \Delta l_i \left[C_{oi}(r_o, B_i/T_i) |p| + \frac{g(B_i/T_i - 2.8)^2 + 4}{(b_i/g)^{3/2} \cdot 10^3} \right] p \quad (A3.7)$$

$$K_{Di} = \frac{1}{2} \rho b_i^4 \Delta l_i \left[C_{oi}(r_o; B_i/T_i) |p| + \frac{72(B_i/T_i - 3.07)^2 + 11}{(b_i/g)^{3/2} \omega^{-4} \cdot 10^3} \right] p \quad (A3.8)$$

$$K_{Di} = \frac{1}{2} \rho b_i^4 \Delta l_i \left[C_{oi}(r_o, B_i/T_i) |p| + \frac{36(B_i/T_i - 2.)^2}{(b_i/g)^{3/2} \omega^{-4} \cdot 10^3} \right] p \quad (A3.9)$$

$$K_{Di} = \frac{1}{2} \rho b_i^4 \Delta l_i \left[.57 |p| + \frac{g(B_i/T_i - 2.82)^2}{(b_i/g)^{3/2} \cdot 10^3} \right] p \quad (A3.10)$$

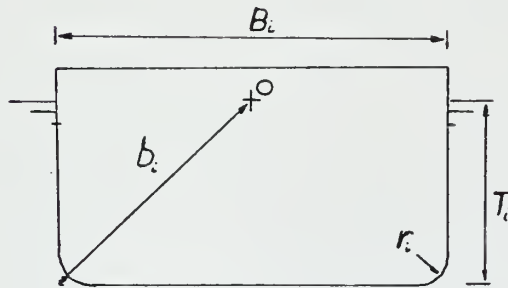


Fig. A3.1

where Eq. A3.7 and A3.8 are used for rectangular sections of different value of $r_o = r_i/b_i$, Eq. A3.9 is used with elliptical sections and Eq A3.10 with terminal section of shape close to triangular, the following values were

found for linear and non linear damping coefficients:

$$\begin{aligned} K_p &= -.00786 \text{ LB*FT*SEC} & - \omega &= 4.145.\text{SEC}(-1) \\ &= -.06835 \text{ LB*FT*SEC} & - \omega &= 8.290.\text{SEC}(-1) \\ K_{p|p|} &= -.01605 \text{ LB*FT*SEC}^2 & - & \text{all frequencies} \end{aligned}$$

The values found in Apenndix 3 for the linear damping coefficients were respectively $-.0022 \text{ LB*FT*SEC}$
 $-.0363 \text{ LB*FT*SEC}$

For the nonlinear heave damping coefficient, the values for each section were obtained by considering each section substituted by an equivalent cylinder of the same diameter when the model moves away from the water (assuming the only drag present to be separation drag) and substituted by a cylinder of the same lenght of wetted contour when the model moves into the water (assuming the only drag present to be skin friction drag) . Data from infinite cylinders was then used, the correspondent values of drag coefficient averaged over a cycle, and the results summed over the lenght of the model. The velocity entered in the calculation of the Reynolds Number was a RMS type of velocity taken from the records of the experiments.

The final value obtained was:

$$Z_w|w| = -1.767 \text{ LB*SEC}^2/\text{FT}^2$$

APPENDIX 4

CIRCUITRY AND DIGITIZING PROGRAM

- 1 - PROGRAM TO DIGITIZE DATA AND CORRESPONDENT
PATCHING IN THE ANALOG COMPUTER


```

C      PROGRAM TO DIGITIZE ROLL DATA
C      INFORMATION RELATED TO SUBPROGRAMS USED IS
C      AVAILABLE IN THE " HYBRID USER'S GUIDE ",
C      PUBLISHED BY THE DEPARTMENT OF MECHANICAL ENGI-
C      NEERING.
C      COMP - LOGICAL FUNCTION WHICH RETURNS A LOGICAL .TRUE. IF
C      THE ADDRESSED COMPARTOR IS IN THE ON STATE, IN THIS
C      PARTICULAR CASE THE COMP. C004.
C      COMP MUST APPEAR ON A LOGICAL STATEMENT
C      LOGICAL COMP
C      DATA ADDR/'C004'/
C      DIMENSION K(3000),A(15)
C      COMPARTOR C004 SHOULD BE IN THE OFF STATE (CLOCK IN STOP,
C      -FOR EXAMPLE ).
C      CLEAR THE ANALOG TO DIGITAL CONVERTER ( ADC )
C      CALL HINT
C      THE COMPUTER IS NOW READY TO ACCEPT THE DATA
C      WRITE(1,200)
C      200 FORMAT( 20?, ' PRESS RUN ON CONSOLE AND START THE TAPE ' )
C      1 IF(.NCT,COMP(ADDR))GO TO 1
C      CALL RBOX(K,0,2,1000)
C      RBOX(K,IFIRST,ILAST,ICNT) - THIS SUBROUTINE READS A BLOCK OF ADC
C      VALUES, EACH TIME THE ADC SAMPLE LINE IS PULSED. THOSE VALUES,
C      IN SCALED-FRACTION FORM ARE STORED IN THE INTEGER ARRAY K.
C      IFIRST AND ILAST ARE, RESPECTIVELY THE FIRST AND LAST CHANNELS
C      FROM WHICH DATA IS READ. ICNT SPECIFIES THE NUMBER OF SAMPLES
C      TO BE READ
C      CALL RBOX(K,0,4,1000)
C      ALL THE DATA IS NOW STORED IN K
C      WRITE(1,600)
C      600 FORMAT(20X,' ADC COMPLETED ' )
C      THE FOLLOWING CARD IS PRESENT TO AVOID THE LAST CARD IN THE
C      PROGRAM TO BE PUNCHED
C      READ(2,100)
C      CONVERT THE DATA FROM SCALED FRACTION INTO REAL, USING THE
C      FUNCTION FLOTS

```



```
DO 3 I=2, 2998
I=I-1
DO 2 J=1, 9
A(J)=FLOTS(K(I))
2 I=I+1
C PUNCH THE DATA ON CARDS
WRITE(2,100)(A(J), J=1,9)
100 FORMAT(9F8.4)
3 CONTINUE
WRITE(1,300)
300 FORMAT( 20X, ' DATA IS DIGITIZED ' )
CALL EXIT
END
```

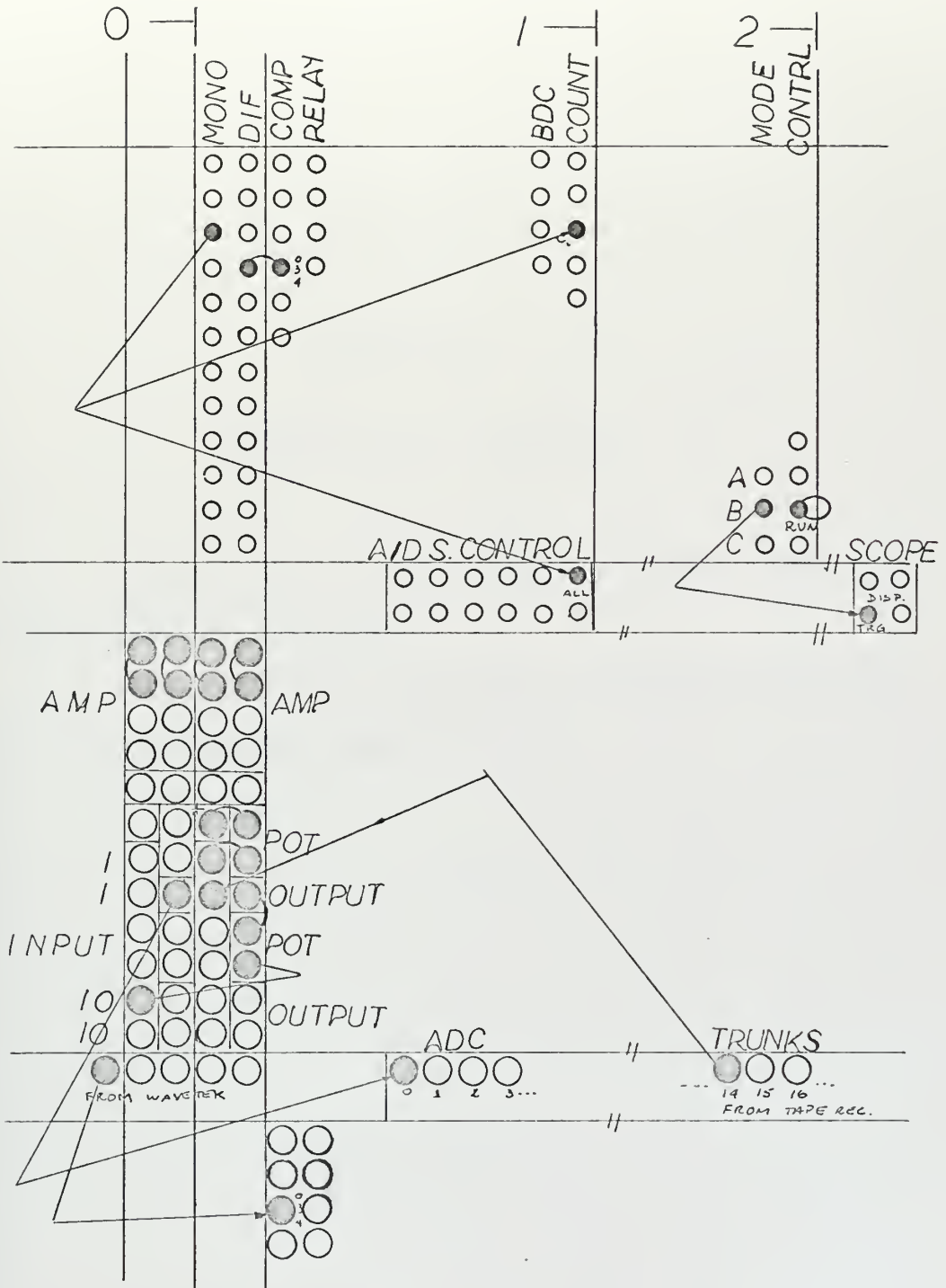



FIG. A4 - 1

EXAMPLE OF PATCHING THE 680 ANALOG COMPUTER

APPENDIX 5

IDENTIFICATION PROCEDURE

- 1 - DISCRIPTION OF THE COMPUTER PROGRAMS USED IN
THE IDENTIFICATION PROCEDURE
- 2 - EXAMPLE OF THE BEHAVIOUR OF VARIABLES AND RE-
LATED VARIANCES DURING IDENTIFICATION


```

C MAIN FOR KALMAN FILTER KLMN
C DISCRPTION OF PARAMETERS
C XHAT - ESTIMATE OF STATE VECTOR X
C Z - MEASUREMENT VECTOR
C U - CONTROL VECTOR
C W - ENVIRONMENTAL NOISE PLUS STRUCTURAL UNCERTAINTY
C V - MEASUREMENT NOISE
C NX - DIMENSION OF THE STATE VECTOR X AND ITS ESTIMATE XHAT
C NZ - DIMENSION OF THE MEASUREMENT VECTOR Z
C NU - DIMENSION OF THE CONTROL VECTOR U
C NTIME - TOTAL NUMBER OF MEASUREMENTS
C DELTA - TIME INTERVAL BETWEEN MEASUREMENTS
C P - COVARIANCE MATRIX OF THE STATE ESTIMATE XHAT
C DQ - COVARIANCE MATRIX OF W ( DIAGONAL )
C DRINV - INVERSE OF THE COVARIANCE MATRIX OF V ( DIAGONAL )
C ZZ - ARRAY FOR THE STORAGE OF THE MEASUREMENTS
C UU - ARRAY FOR STORAGE OF CONTROL
C Q = Sqrt(DQ/DELTA)
C R = Sqrt(DELTA/DRINV)
C YPLOT - EACH OF XHAT,U AND Z FOR THE PURPOSE OF PLOTTING
C UCAL - SCALING OF THE AMPLITUDE OF THE CONTROL FOR
C THE PURPOSE OF PLOTTING
C SIMBL(K) - LABELS CF YPLOT IN THE PLOT
C YMIN,YMAX - SCALING OF YPLOT FOR THE PURPOSE OF PLOTTING
C NUMBR - NUMBER OF DESIRED RUNS ON THE DATA
C FOR EACH RUN IT IS NECESSARY TO SPECIFY
C A SET OF VALUES OF Q(I), R(I), P(I,I) AND XHAT(I)
C
C FORMAT OF INPUT PARAMETERS
C
C CARD 1 NUMBR FORMAT(I2)
C CARD 2 NX,NU,NZ,NTIME,DELTA FORMAT(3I2,I4,F10.5)
C CARD 3 UCAL,YMIN,YMAX FORMAT(3F10.5)
C CARD 4 SYMBJ(K),K=1, (NX+NU+NZ) FORMAT(20A1)
C CARD 5 (UU(I,J),ZZ(K,J), I=1, NU, K=1,NZ),J=1 NTIME

```



```

C
C
C      CARD M   Q(I), I=1, NX          FORMAT(9F8.4)
C      CARD M+1 R(I), I=1, NX          FORMAT(10F10.5)
C      CARD M+2 P(I,I), I=1, NX        FORMAT(10F10.5)
C      CARD M+3 XHAT(I), I=1, NX       FORMAT(10F10.5)
C      DIMENSION Z(10),XHAT(10),U(10),P(10,10),DQ(10),DRINV(10),Y(10),Q(1
    .10),R(10),YPLOT(20),SYMBL(20),ZZ(10,1000),UU(10,1000)
C      READ(5,1000)NUMBR
C      1000 FORMAT(I2)
C      READ(5,2000)NX,NU,NZ,NTIME,DELTA
C      2000 FORMAT(3I2,I4,F10.5)
C      NY=NX+NZ+NU
C      READ(5,6002)UCAL,YMIN,YMAX
C      6002 FORMAT(3F10.5)
C      READ(5,6003)(SYMBL(K),K=1, NY)
C      6003 FORMAT(20A1)
C      JJ = J-1, NTIME
C      READ(5,3000)((UU(I,J),I=1, NL),(ZZ(I,J),I=1, NZ))
C      3000 FORMAT(9F8.4)
C      4 CONTINUE
C      DO 1 NN=1, NUMBR
C      DO 11 I=1,NX
C      DO 11 J=1,NX
C      11 P(I,J)=0.0
C      READ(5,6001)(Q(I),I=1, NX)
C      READ(5,6001)(R(I),I=1, NZ)
C      READ(5,6001)(P(I,I),I=1, NX)
C      READ(5,6001)(XHAT(I),I=1, NX)
C      6001 FORMAT(10F10.5)
C      WRITE(6,201)NX,NU,NZ,NTIME,DELTA
C      WRITE(6,202)(Q(I),I=1, NX)
C      WRITE(6,203)(R(I),I=1, NZ)
C      WRITE(6,204)(P(I,I),I=1, NX)
C      WRITE(6,205)(XHAT(I),I=1, NX)
C      201 FORMAT(10X,'NX = ',I2/10X,'NU = ',I2/10X,'NZ = ',I2/7X,'NTIME = ',

```



```

1 I4/7X,'DELTA = ',F10.5)
202 FORMAT (5H Q ,6F10.4)
203 FORMAT (5H R ,6F10.4)
204 FORMAT (5H P ,6F10.4)
205 FORMAT (5H XHAT,6F10.4)
      CALL PRPLT(YPLOT,SYMBL,YMIN,YMAX,NY,0)
      DO 12 I=1,NX
12 DQ(I)=DELTA*Q(I)*Q(I)
      DO 14 I=1,NZ
14 DRINV(I)=DELTA/(R(I)*R(I))
      DO 27 NT=1,NTIME
      DO 2 I=1, NU
2 U(I)=UU(I,NT)
      DO 3 I=1, NZ
3 Z(I)=ZZ(I, NT)
      CALL KLMN(Z,XHAT,U,P,DQ,DRINV,NX,NZ,DETP1,DETP2,DELTA)
      IF(DETP1)25,13,25
25 IF(DETP2)26,13,26
26 K=0
      DO 41 I=1,NX
      K=K+1
41 YPLOT(K)=XHAT(I)
      DO 42 I=1,NZ
      K=K+1
42 YPLOT(K)=Z(I)
      DO 43 I=1,NU
      K=K+1
43 YPLOT(K)=UCAL*U(I)
      CALL PRPLT(YPLOT,SYMBL,YMIN,YMAX,NY,1)
      IF(NT-20*(NT/20))27,47,27
47 WRITE(7,110)NT
      WRITE(7,101)(XHAT(I), I=1, NX)
      WRITE(7,103)(Z(I), I=1, NZ)
      WRITE(7,104)(P(I,I),I=1, NX)
27 CONTINUE
      WRITE(7,100)

```



```
DO 31 I=1,NX
  31 WRITE(7,104)(P(I,J), J=1, NX)
  100 FORMAT (1H0)
  101 FORMAT (6H XHAT ,10F10.3)
  103 FORMAT (6H Z ,10F10.3)
  104 FORMAT (6H P ,10F10.3)
  110 FORMAT (4H NT=,I3)
  113 WRITE(7,4000)DETP1,DETP2
4000 FORMAT(2F10.5)
  1 CONTINUE
  CALL EXIT
END
```



```

C SUBROUTINE KLMN(Z,X,U,P,CQ,DRINV,NX,NZ,DETP1,DETP2,DELTA)
C KALMAN FILTER
C DISCRPTION OF PARAMETERS
C THE SAME AS IN MAIN PROGRAM, EXCEPT:
C PROVIDED BY CALLING PROGRAM
C X - ESTIMATE OF THE STATE BEFORE MEASUREMENT
C P - COVARIANCE OF THE STATE BEFORE MEASUREMENT
C RETURNED BY KLMN
C X - ESTIMATE OF THE STATE AFTER MEASUREMENT
C P - COVARIANCE OF THE STATE AFTER MEASUREMENT
C DETP1 - DETERMINANT OF THE MATRIX TO BE INVERTED
C IF DETP1 = 0 - SOLUTION IS SINGULAR
C DIMENSION Z(10),X(10),U(10),P(10,10),
C 10 Q(10),DRINV(10),F(10,10),D(10,10),Y(10)
C CALL FOFXU(X,U,Y)
C DO 10 I=1,NX
C 10 X(I)=X(I)+DELTA*Y(I)
C CALL FDERV(F,X,U,NX)
C DO 12 I=1,NX
C DO 12 J=1,NX
C SUM=0.0
C DO 11 K=1,NX
C 11 SUM=SUM+F(I,K)*P(K,J)
C 12 C(I,J)=DELTA*SUM
C DO 15 I=1,NX
C DO 14 J=1,NX
C P(I,J)=P(I,J)+D(I,J)+D(J,I)
C 14 P(J,I)=P(I,J)
C 15 P(I,I)=P(I,I)+DQ(I)
C DO 17 I=1,NZ
C DO 16 J=1,NZ
C C(I,J)=P(I,J)
C 17 D(I,I)=D(I,I)+1.0/DRINV(I)
C CALL MINIO(O,NZ,DETP1)
C DO 23 I=1,NX
C DO 23 J=1,NZ

```



```
SUM=0.0
DO 22 K=1,NZ
22 SUM=SUM+P(I,K)*D(K,J)
23 F(I,J)=SUM
DO 25 I=1,NX
DO 25 J=1,NX
SUM=0.0
DO 24 K=1,NZ
24 SUM=SUM+F(I,K)*P(K,J)
25 C(I,J)=SUM
DO 26 I=1,NX
DO 26 J=1,NX
26 P(I,J)=P(I,J)-D(I,J)
GETP2=1.0
18 DO 19 I=1,NZ
19 Y(I)=Z(I)-X(I)
DO 20 I=1,NX
DO 20 K=1,NZ
20 X(I)=X(I)+P(I,K)*Y(K)+DRINV(K)
13 CONTINUE
RETURN
END
```



```

C      SUBROUTINE PRPLT(Y,SYMBL,YMIN,YMAX,NY,NEWPL)
C      PLOTTING ROUTINE
C      PARAMETERS AS IN MAIN, EXCEPT:
C      Y = YPLOT
C      NY = NX + NZ + NU
C      NEWPL = CONTROL VARIABLE
C      IF NEWPL = 0 ROUTINE SCALES AND LABELS THE AXIS
C      IF NEWPL = 1 ROUTINE PLOTS
C      DIMENSION A(101),Y(20),SYMBL(20)
C      DATA BLANK/' ',ASTER/'*'/
C      IF(NEWPL)21,11,21
C      11 NPRNT=101
C      NP10=(NPRNT-1)/10+1
C      NHALF=10*(NPRNT-1)/20+1
C      SCALE=NPRNT-1
C      SCALE=SCALE/(YMAX-YMIN)
C      N=0
C      DO 12 I=1,NP10
C      XI=10*(I-1)
C      12 A(I)=YMIN+XI/SCALE
C      PRINT 101
C      101 FORMAT (1H1)
C      PRINT 112, (A(I),I=1,NP10)
C      112 FORMAT (11E10.2)
C      DO 14 I=1,NPRNT
C      14 A(I)=BLANK
C      DO 15 I=1,NP10
C      J=10*(I-1)+1
C      15 A(J)=ASTER
C      PRINT 114, (A(I),I=1,NPRNT)
C      111 FORMAT (5X,101A1)
C      RETURN
C      21 N=N+1
C      N5=N-5*(N/5)
C      DO 22 I=1,NPRNT
C      22 A(I)=BLANK

```



```
A(1)=ASTER
A(NHALF)=ASTER
A(NPRNT)=ASTER
IF(N5)25,23,25
23 DO 24 I=1,NP10
   J=10*(I-1)+1
24 A(J)=ASTER
25 DO 26 K=1,NY
   IF(Y(K)-YMIN)28,26,26
26 IF(Y(K)-YMAX)27,27,23
27 J=SCALE*(Y(K)-YMIN)+1.5
   A(J)=SYMBL(K)
28 CONTINUE
   IF(N5)41,42,41
41 PRINT 111, (A(I),I=1,NPRNT)
   RETURN
42 PRINT 113, N,(A(I),I=1,NPRNT)
113 FORMAT (1X,I3,1X,101A1)
   RETURN
   END
```



```

C      SUBROUTINE MINIO(A,N,D)
      MATRIX INVERSION
      DIMENSION A(10,10),L(10),M(10)
      SEARCH FOR LARGEST ELEMENT
      D=1.0
      DO 80 K=1,N
      L(K)=K
      M(K)=K
      BIGA=A(K,K)
      DO 20 J=K,N
      DO 20 I=K,N
      10 IF (ABS(BIGA)-ABS(A(I,J)))15,20,20
      15 BIGA=A(I,J)
      L(K)=I
      M(K)=J
      20 CONTINUE
      INTERCHANGE ROWS
      J=L(K)
      IF (J-K)35,35,25
      25 DO 30 I=1,N
      HOLD=-A(K,I)
      A(K,I)=A(J,I)
      30 A(J,I)=HOLD
      INTERCHANGE COLUMNS
      I=M(K)
      IF (I-K)45,45,38
      38 DO 40 J=1,N
      HOLD=-A(J,K)
      A(J,K)=A(J,I)
      40 A(J,I)=HOLD
      DIVIDE CLOUMN BY MINUS PIVOT
      45 IF (ABS(BIGA)-1.0E-20)46,46,48
      46 D=0.0
      RETURN
      48 DO 55 I=1,N
      IF (I-K)50,55,50
      MINVD 3
      MINVD 4
      MINVD 5
      MINVD 6
      MINVD 7
      MINVD 8
      MINVD 9
      MINVD 10
      MINVD 11
      MINVD 12
      MINVD 13
      MINVD 14
      MINVD 15
      MINVD 16
      MINVD 17
      MINVD 18
      MINVD 19
      MINVD 20
      MINVD 21
      MINVD 22
      MINVD 23
      MINVD 24
      MINVD 25
      MINVD 26
      MINVD 27
      MINVD 28
      MINVD 29
      MINVD 30
      MINVD 31
      MINVD 32
      MINVD 33
      MINVD 34
      MINVD34A

```



```

50 A(I,K)=A(I,K)/(-BIGA)
55 CONTINUE
C    REDUCE MATRIX
  DO 65 I=1,N
    HOLD=A(I,K)
    DO 65 J=1,N
      IF(I-K)60,65,60
      IF(J-K)62,65,62
      62 A(I,J)=HOLD*A(K,J)+A(I,J)
      65 CONTINUE
C    DIVIDE ROW BY PIVOT
  DO 75 J=1,N
    IF(J-K)70,75,70
    70 A(K,J)=A(K,J)/BIGA
    75 CONTINUE
C    PRODUCT OF PIVOTS
  D=D*BIG
C    REPLACE PIVOT BY RECIPROCAL
  A(K,K)=1.0/BIG
  30 CONTINUE
C    FINAL ROW AND COLUMN INTERCHANGE
  K=N
  100 K=K-1
    IF(K)150,150,105
    105 I=L(K)
    IF(I-K)120,120,108
    108 DO 110 J=1,N
      HOLD=A(J,K)
      A(J,K)=-A(J,I)
      110 A(J,I)=HOLD
      120 J=M(K)
    IF(J-K)100,100,125
    125 DO 120 I=1,N
      HOLD=A(K,I)
      A(K,I)=-A(J,I)
      130 A(J,I)=HOLD

```

MINVD 35
 MINVD 36
 MINVD 37
 MINVD 38
 MINVD 39
 MINVD 40
 MINVD 41
 MINVD 42
 MINVD 43
 MINVD 44
 MINVD 45
 MINVD 46
 MINVD 47
 MINVD 48
 MINVD 49
 MINVD 50
 MINVD 51
 MINVD 52
 MINVD 53
 MINVD 54
 MINVD 55
 MINVD 56
 MINVD 57
 MINVD 58
 MINVD 59
 MINVD 60
 MINVD 61
 MINVD 62
 MINVD 64
 MINVD 65
 MINVD 66
 MINVD 67
 MINVD 68
 MINVD 69
 MINVD 70

GO TO 100
150 RETURN
END

MINVD 71
MINVD 72
MINVD 73


```

C
C SUBROUTINE FOFXU(X,U,Y)
C NONLINEAR EQUATIONS OF MOTION - HEAVE
C FOR THE PURPOSE OF IDENTIFYING BOTH LINEAR AND NONLINEAR COEFFI-
C CIENTS
C DISCRPTION OF PARAMETERS
C Y(I) - NONDIMENSIONAL TIME DERIVATIVE OF X(I)
C Y(1) = WDOT/((DRAFT/2.)*OMEGA**2)
C Y(2) = ZDOT/((DRAFT/2.)*OMEGA)
C Y(3) = 0.0
C Y(4) = 0.0
C Y(5) = 0.0
C Y(6) = 0.0
C X(I) - NONDIMENSIONAL STATE VECTOR
C X(1) = W/((DRAFT/2.)*OMEGA)
C X(2) = Z/(DRAFT/2)
C X(3) = (Z(W)/(-Z(WDOT)+M))* (1./OMEGA)
C X(4) = (Z(Z)/(-Z(WDOT)+M))* (1./OMEGA**2)
C X(5) = 10.5*(.5*Z(W*ABS(W))/(-Z(WDOT)+M))* (DRAFT/2.)
C X(6) = 10.5*(.5*Z(Z**2)/(-Z(WDOT)+M))* ((DRAFT/2.)/OMEGA**2)
C U(I) - NONDIMENSIONAL CONTROL
C U(1) = ZETA/(DRAFT/2.)
C INITIAL CONDITIONS - CMEGA = 8.038 SEC(-1)
C X(1) = 0.0
C X(2) = 0.0
C X(3) = -.347
C X(4) = -1.129
C X(5) = -.930
C X(6) = -.585
C
C DIMENSION Y(10),X(10),U(10)
C Y(1)=X(3)*X(1)+X(4)*X(2)-U(1)+0.1*X(5)+0.1*ABS(X(1))+0.1*X(6)*X
C (2)-U(1)**2
C Y(2)=X(1)
C Y(3)=0.0
C Y(4)=0.0
C Y(5)=0.0
C Y(6)=0.0
C RETURN
C END

```



```

C
C
C
C
C
C
C
C
C
C
SUBROUTINE FDERV(F,X,U,NX)
PARTIAL DERIVATIVE OF FOFXU WITH RESPECT TO THE STATE VECTOR X
FOR THE EQUATIONS PRESENTED ABOVE
DISCUSSION OF PARAMETERS
  F(I,J) = DERIVATIVE OF Y(I) WITH RESPECT TO X(J)
  X = STATE VECTOR
  U = CONTROL VECTOR
  NX = DIMENSION OF THE STATE VECTOR

DIMENSION F(10,10),X(10),U(10)
DO 1 J=1, NX
DO 1 J=1, NX
  1 F(I,J)=0.0
  F(1,1)=X(3)+0.2*X(5)*ABS(X(1))
  F(1,2)=X(4)+0.2*X(6)*X(2)-U(1)
  F(1,3)=X(1)
  F(1,4)=X(2)-U(1)
  F(1,5)=0.1*X(1)*ABS(X(1))
  F(1,6)=0.1*(X(2)-U(1))**2
  F(2,1)=1.0
RETURN
END

```



```

SUBROUTINE FOFXU(X,U,Y)
C
C NONLINEAR EQUATIONS OF MOTION - ROLL
C FOR THE PURPOSE OF IDENTIFYING BOTH LINEAR AND NONLINEAR COEFFI-
C DISCUSSION OF PARAMETERS
C
C Y(I) - NONDIMENSIONAL TIME DERIVATIVE OF X(I)
C
C Y(1) = 100*(PDDT/OMEGA**2)
C Y(2) = 100*(FIDOT/OMEGA)
C Y(3) = 0.0 FOR I > 2
C
C X(I) - NONDIMENSIONAL STATE VECTOR
C
C X(1) = 100*(P/OMEGA)
C X(2) = 100*(FI)
C X(3) = 100*(K(P)/(-K(PDOT)+I(XX))/OMEGA)
C X(4) = (K(FI)/(-K(PDOT)+I(XX))/OMEGA**2)
C X(5) = 0.1*((WEIGHT*BG)/(-K(PDOT)+I(XX)))/OMEGA**2)
C X(6) = (K(P*ABS(P))/(-K(PDOT)+I(XX)))
C X(7) = (0.167*(K(FI**3)/(-K(PDOT)+I(XX)))/OMEGA**2)
C
C U(I) - NONDIMENSIONAL CONTROL
C
C U(1) = WAVE SLOPE
C
C INITIAL CONDITIONS - OMEGA = 4.145 SEC(-1)
C
C X(1) = 0.0
C X(2) = 0.0
C X(3) = -0.420
C X(4) = -0.989
C X(5) = 0.274
C X(6) = -0.120
C X(7) = 0.512
C
C
C DIMENSION Y(10),X(10),U(10)
C
C Y(1)=0.01*X(3)+X(1)+(X(4)-10.0*X(5))*(X(2)-U(1))+10.0*X(5)+X(2)+0.
C 11*X(6)+X(1)+ABS(X(1))+0.1*(X(7)+0.1667*X(5))*(X(2)-U(1))+0.0167
C 1*X(5)+X(2)**3
C
C Y(2)=X(1)
C Y(3)=0.0
C Y(4)=0.0
C Y(5)=0.0
C Y(6)=0.0

```


Y(7)=0.0
RETURN
END


```

C SUBROUTINE FDERV (F,X,U,NX)
C PARTIAL DERIVATIVE OF FOFXU WITH RESPECT TO THE STATE VECTOR X
C FOR THE EQUATIONS PRESENTED ABOVE
C DISCUSSION OF PARAMETERS
C F(I,J) - DERIVATIVE OF Y(I) WITH RESPECT TO X(J)
C X - STATE VECTOR
C U - CONTROL VECTOR
C
C DIMENSION F(10,10),X(10),U(10)
C DO 1 I=1, NX
C DO 1 J=1, NX
C 1 F(I,J)=0.0
C F(1,1)=0.01*X(3)+0.2*X(5)*ABS(X(1))
C F(1,2)=X(4)+(0.3*X(7)+0.05*X(5))*X(2)-U(1)*X(2)*X(2)
C F(1,3)=0.01*X(1)
C F(1,4)=X(2)-U(1)
C F(1,5)=-1.0+U(1)+0.01667*X(2)-U(1)*X(2)*X(2)
C F(1,6)=0.1*X(1)*ABS(X(1))
C F(1,7)=0.1*(X(2)-U(1))*X(2)
C F(2,1)=1.0
C RETURN
C END

```



```

SUBROUTINE FOFXU(X,U,Y)
NONLINEAR EQUATIONS OF MOTION - ROLL
FOR THE PURPOSE OF IDENTIFYING ONLY THE LINEAR TERMS IN THE
EQUATIONS OF MOTION
DISCUSSION OF PARAMETERS
THE PARAMETERS ARE THE SAME AS FOR THE GENERAL CASE, EX-
CEPT THAT IN THESE EQUATIONS X(6) AND X(7) ARE SUBSTITU-
TED BY THEIR PREDICTED VALUE
DIMENSION Y(10),X(10),U(10)
Y(1)=0.01*X(3)*X(1)+(X(4)-10.0)*X(5))*X(2)-U(1))+10.0*X(5)*X(2)-0.
10126*X(1)*ABS(X(1))+(0.0312+0.0167*X(5))*X(2)-U(1))*X(2)*X(5
1)*X(2)*X(3
Y(2)=X(1)
Y(3)=0.0
Y(4)=0.0
Y(5)=0.0
RETURN
END

```



```
C
C
SUBROUTINE FDERV(F,X,U,NX)
PARTIAL DERIVATIVE OF FOFXU WITH RESPECT TO THE STATE VECTOR X
FOR THE EQUATIONS PRESENTED ABOVE
DIMENSION F(10,10),X(10),U(10)
DO 1 I=1, NX
DO 1 J=1, NX
1 F(I,J)=0,0
F(1,1)=0,01 X(3)-0,0252*ABS(X(2))
F(1,2)=X(4)+(0,0936+0,05*X(5))*X(2)-U(1)*X(5)*X(2)*X(2)
F(1,3)=0,01*X(1)
F(1,4)=X(2)-U(1)
F(1,5)=10,0*U(1)+0,0167*(X(2)-U(1))*X(3)-0,0167*X(2)*X(3)
F(2,1)=1,0
RETURN
END
```



```
C
SUBROUTINE FOFXU(X,U,Y)
  DIMENSION Y(10),X(10),U(10)
  LINEAR EQUATIONS OF MOTION - ROLL
  Y(1)=0.01*X(3)*X(1)+(X(4)-10.0*X(5))*X(2)-U(1))+10.0*X(5)*X(2)
  Y(2)=X(1)
  Y(3)=0.0
  Y(4)=0.0
  Y(5)=0.0
  RETURN
END
```



```
C
C
SUBROUTINE FDERV (F,X,U,NX)
PARTIAL DERIVATIVE OF FOFXU WITH RESPECT TO THE STATE VECTOR X
FOR THE EQUATIONS PRESENTED ABOVE
DIMENSION F(10,10),X(10),U(10)
DO 1 I=1, NX
  DO 1 J=1, NX
    1 F(I,J)=0.0
    F(1,1)=0.01*X(3)
    F(1,2)=X(4)
    F(1,3)=0.01*X(1)
    F(1,4)=X(2)-U(1)
    F(1,5)=10.0*X(1)
    F(2,1)=1.0
  RETURN
END
```



```

ROUTINES OF DIFFERENT SET UP FOR THE PURPOSE
SIMULATION
C
C MAIN FOR KLMN
C
C PARAMETERS AS ABOVE, EXCEPT:
C XX - TRUE VALUE OF THE STATE VECTOR X
C COEFW, COEFV - CONTROL PARAMETERS FOR THE AMOUNT
C OF NOISE PRESENT, USUALLY 0 < COEFW, COEFV < 1
C THERE ARE NO STORAGE ARRAYS ZZ AND UU
C
C FORMAT OF INPUT PARAMETERS
C
C CARD 1  NUMBER          FORMAT(I2)
C CARD 2  NX,NU,NZ,NTIME,DELTA  FORMAT(3I2,I4,F10.5)
C CARD 3  UCAL,YMIN,YMAX        FORMAT(3F10.5)
C CARD 4  SYMBJ(K),K=1, (NX+NU+NZ)  FORMAT(20A1)
C CARD 5  O(I),I=1, NX          FORMAT(10F10.5)
C CARD 6  R(I),I=1,NZ           FORMAT(10F10.5)
C CARD 7  P(I),I=1, NX          FORMAT(10F10.5)
C CARD 8  XHAT(I),I=1,NX        FORMAT(10F10.5)
C CARD 9  XX(I),I=1, NX         FORMAT(10F10.5)
C CARD 10 COEFW,COEFV           FORMAT(10F10.5)
C
C DIMENSION Z(10),XHAT(10),U(10),P(10,10),DQ(10),DRINV(10),
C 1Y(10),XX(10),W(10),V(10),Q(10),R(10),YPLCT(20),SYMBL(20)
C
C READ(5,1000)NUMBER
C 1000  FORMAT(I2)
C
C READ(5,2000)NX,NU,NZ,NTIME,DELTA
C 2000  FORMAT(3I2,I4,F10.5)
C
C NY=NX+NZ+NU
C
C READ(5,6002)UCAL,YMIN,YMAX
C 6002  FORMAT(3F10.5)
C
C READ(5,6003)(SYMBL(K),K=1, NY)
C 6003  FORMAT(20A1)
C
C DO 1 NN=1, NUMBER
C DO 11 I=1,NX
C DO 11 J=1,NX
C 11  P(I,J)=0.0
C
C READ(5,6001)COEFW,COEFV

```



```

READ(5,6001)(Q(I),I=1, NX)
READ(5,6001)(R(I),I=1, NZ)
READ(5,6001)(P(I),I=1, NX)
READ(5,6001)(XHAT(I),I=1, NX)
READ(5,6001)(XX(I),I=1, NX)
6001 FORMAT(10F10.5)
WRITE(6,200)COEFW,COEFV
WRITE(6,201)NX,NU,NZ,NTIME,DELTA
WRITE(6,202)(Q(I),I=1, NX)
WRITE(6,203)(R(I),I=1, NZ)
WRITE(6,204)(P(I),I=1, NX)
WRITE(6,205)(XHAT(I),I=1, NX)
WRITE(6,206)(XX(I),I=1, NX)
200 FORMAT(10X,' COEFW = ',F10.5/10X,' COEFV = ',F10.5)
201 FORMAT(10X,'NX = ',12/10X,'NU = ',12/10X,'NZ = ',12/7X,'NTIME = ',
114/7X,'DELTA = ',F10.5)
202 FORMAT(5H Q ,6F10.4)
203 FORMAT(5H R ,6F10.4)
204 FORMAT(5H P ,6F10.4)
205 FORMAT(5H XHAT,6F10.4)
206 FORMAT(5H XX ,6F10.4)
CALL PRPLOT(YPLOT,SYMBL,YMIN,YMAX,NY,0)
DO 12 I=1,NX
QQ(I)=DELTA*Q(I)*Q(I)
12 Q(I)=COEFW*Q(I)/SQRT(DELTA)
DO 14 I=1,NZ
SRINV(I)=DELTA/(R(I)*R(I))
14 R(I)=COEFV*R(I)/SQRT(DELTA)
DO 27 NT=1,NTIME
DO 22 I=1,NZ
W(I)=Q(I)*RAND(-1.0)
22 V(I)=R(I)*RAND(-1.0)
CALL CNTRL(U,NT)
CALL FOFXU(XX,U,Y)
DO 23 I=1,NZ
23 XX(I)=XX(I)+DELTA*(Y(I)+W(I))

```



```

DO 24 I=1,NZ
24  Z(I)=XX(I)+V(I)
    CALL KLMN(Z,XHAT,U,P,DQ,DRINV,NX,NZ,DETP1,DETP2,DELTA)
    IF(DETP1)25,13,25
25  IF(DETP2)26,13,26
26  K=0
    DO 41 I=1,NX
    K=K+1
41  YPLOT(K)=XHAT(I)-XX(I)
    DO 42 I=1,NZ
    K=K+1
42  YPLOT(K)=Z(I)-XX(I)
    DO 43 I=1,NU
    K=K+1
43  YPLOT(K)=UCAL*U(I)
    CALL PRPLT(YPLOT,SYMBL,YMIN,YMAX,NY,1)
    IF(NT-20*(NT/20))27,47,27
47  WRITE(7,110)NT
    WRITE(7,101)(XHAT(I), I=1, NX)
    WRITE(7,102)(XX(I), I=1, NX)
    WRITE(7,103)(Z(I), I=1, NZ)
    WRITE(7,104)(P(I,1),I=1, NX)
27  CONTINUE
    WRITE(7,100)
    DO 31 I=1,NX
31  WRITE(7,104)(P(I,J), J=1, NX)
100  FORMAT (1H0)
101  FORMAT (6H XHAT ,10F10.3)
102  FORMAT (6H XX ,10F10.3)
103  FORMAT (6H Z ,10F10.3)
104  FORMAT (6H P ,10F10.3)
110  FORMAT (4H NT=,I3)
13  WRITE(7,1000)DETP1,DETP2
1000  FORMAT(2F10.5)
    CALL EXIT
    END

```



```
C      FUNCTION RAND(X)
      RANDOM NUMBER GENERATOR
      IF(X)3,2,1
1     IX=X
2     RAND=0.0
      N=1
      GO TO 4
3     RAND=-6.0
      N=12
4     N=N-1
      IX=IX+65539
      IF(IX)5,6,6
5     IX=IX+2147483647+1
      Y=IX
      RAND=RAND+Y*.4656613E-9
      IF(N)7,7,4
7     CONTINUE
      RETURN
      END
```



```
C
SUBROUTINE CNTRL(U,NT)
  INPUT GENERAT6R
  DIMENSION U(10)
  TT=NT
  U(1)=0.2987*SIN(6.28318*TT/20.0)
  RETURN
END
```

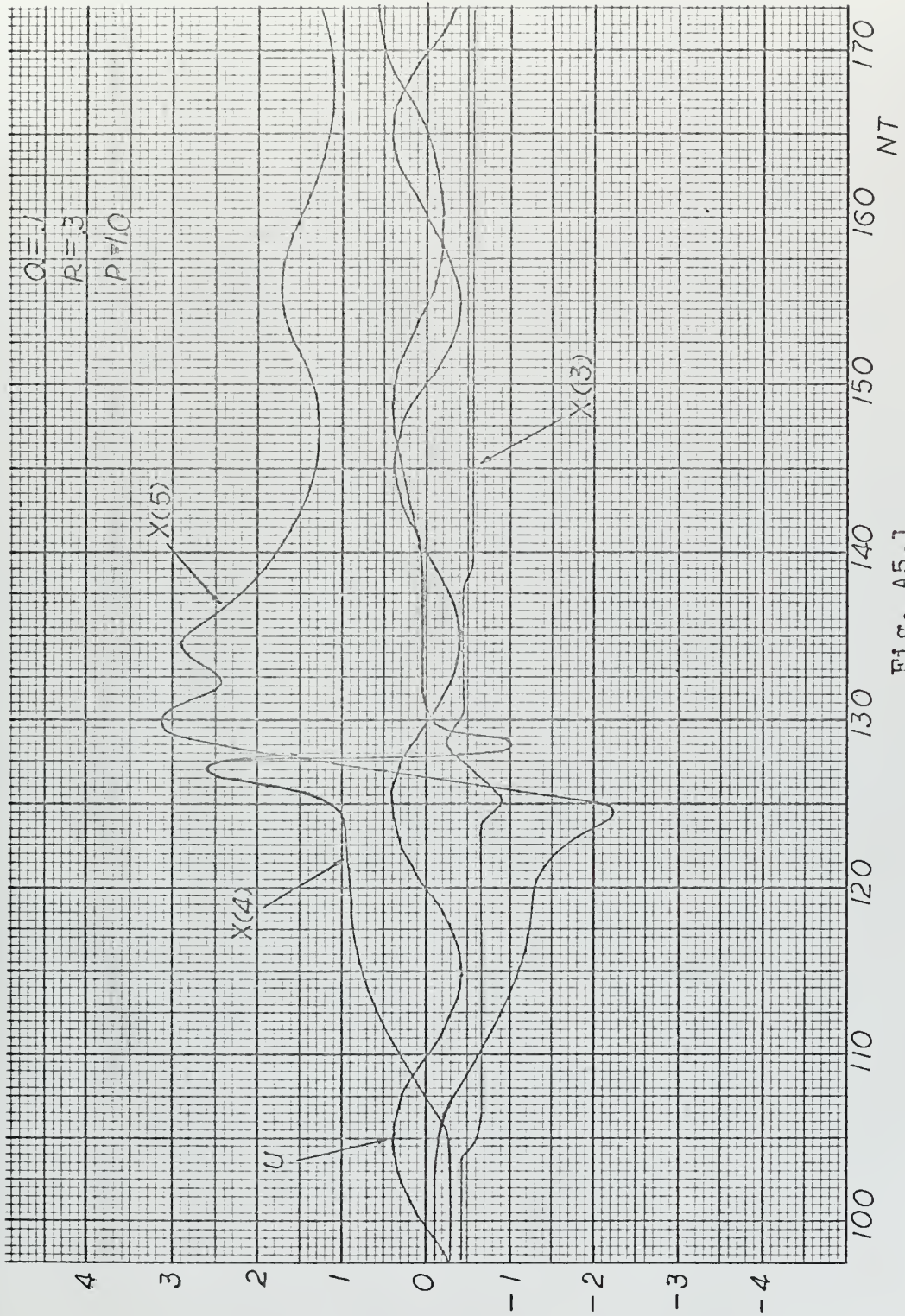



Fig. A5.1.

Typical behaviour of the parameters to be identified



26 APR 72

18816
25913

127307

Thesis
R3299

Reis

Identification of
ship model motion
parameters.

21 SEP 71
26 APR 72

DISPLAY
18816
25913

Thesis
R3299

Reis

Identification of
ship model motion
parameters.

127307

thesR3299

Identification of ship model nation para



3 2768 002 02319 4

DUDLEY KNOX LIBRARY